

Projectile Motion

Objective

To study the trajectory of a low speed projectile. Specifically to compare an experimentally measured point, such as the point where the projectile strikes the floor to the point where it is predicted to strike the floor.

Pre-Lab Assignment

This experiment involves motion in a plane (with x - and y -components). Therefore, you will be working with two sets of equations for each of the two orthogonal directions (x and y), with time, a scalar, the same for both directions. The following exercise will help you to understand the physics behind this experiment.

A ball is fired in the horizontal direction from a spring gun (mounted on a table of height $H = 1.0$ m) and strikes the floor $R = 0.70$ m away from the foot of the table (Fig. 1.)

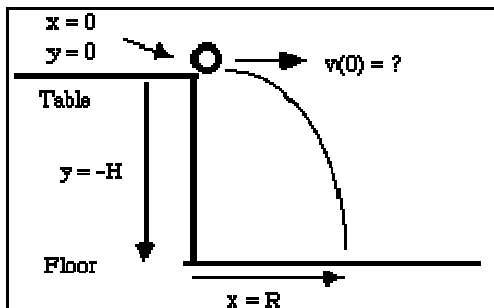


Fig. 1. Trajectory of a Projectile

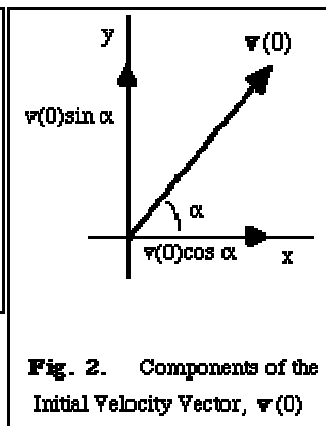


Fig. 2. Components of the Initial Velocity Vector, $v(0)$

- Find the time it takes for the ball to strike the floor.
- Find the muzzle velocity (as it leaves the spring gun) of the ball. What is the velocity vector when the ball strikes the floor? Is its magnitude (speed) the same as the muzzle speed?
- Next assume that the spring gun is mounted at an angle (Fig. 2.) of $\theta = 30^\circ$ to the horizontal direction and the ball leaves the spring gun with the same muzzle speed. Compute 10 points, each with (x, y) , on the trajectory and draw a graph. Where will the ball land on the floor? You may use graphic software to make the plot. A good choice is either Excel or Cricket Graph.

Introduction

This study is concerned with the motion of objects in the gravitational field near the surface of the Earth where the field is very nearly uniform, therefore, the acceleration due to gravity, g , and the objects weight, $W = mg$, are constants. In addition, the object's range and vertical motion is limited, so that the effects of the Earth's rotation can be neglected. (These effects are important in long range artillery and rocket flight.) Also, for dense objects of sufficient size and moving at low speed, one can neglect the effect of air resistance. Thus the object moves under the influence of a single force, its weight, which is constant in both magnitude and direction. The following analysis makes use of this circumstance.

Theory

In accordance with Newton's 2nd Law, an object experiences a constant acceleration when subject to a single constant force, here assumed to be the object's weight. The direction of the acceleration is the same as the direction of the force (here "down" or along $-y$), and the magnitude of the acceleration is calculated from: $a = F/m = mg/m = 9.80 \text{ m/s}^2$. If the initial velocity of the object is zero, or if the initial velocity lies along the line of the acceleration, then the motion is rectilinear. If, however, the initial velocity has a component perpendicular to the acceleration vector, i.e., an x -component, then we have what is called projectile motion. The motion can be analyzed by vector components: In the vertical direction the motion is uniformly decelerated whenever the y -component of velocity is + (against g) and accelerated whenever the y -component of velocity is - (with g). Both cases are described by Eq. (1) below. In the horizontal direction no forces act so we have uniform motion (acc. = 0) described by Eq. (2).

$$y = v_{oy} t - (1/2)gt^2 = v_o \sin\theta t - (1/2)gt^2 \quad (1)$$

$$x = v_{ox} t = v_o \cos\theta t \quad (2)$$

These equations are parametric in time. Eliminating t we get from (2) $t = x/v_o \cos\theta$ and so:

$$y = v_o \sin\theta (x / v_o \cos\theta) - (1/2)g (x^2 / v_o^2 \cos^2\theta)$$

or

$$y = (\tan\theta) x - (g / 2v_o^2 \cos^2\theta) x^2 \quad (3)$$

This equation is of the form $y = Bx + Cx^2$, i.e., a parabola, and since the constant, C , is negative for all values of a , the parabola opens downward. When $a = 90^\circ$ or 270° the parabola degenerates into a straight line, i.e., rectilinear motion.

Name _____

Class _____ Date _____

Apparatus

Spring gun with one projectile

“Catcher” box

blank paper

carbon paper

tape

2-meter stick.

Procedure

1. Inspect the spring gun. Note that there are three loading positions. Choose a loading position by making test firings at approximately 40° to 70° and keeping the projectile from striking the ceiling or the far wall. Use that loading position throughout the experiment. This maintains the same muzzle speed for the entire experiment.
2. Always leave the gun unloaded or uncocked until just before you are ready to fire for safety reasons and exercise reasonable caution for safety when firing.
3. Set the gun in the horizontal position ($\alpha = 0$) and measure and record H (the distance from the bottom of the ball projectile to the floor). Fire the gun into the catcher equipped with carbon paper covered and taped at the leading edge with plane paper. A properly positioned catcher will record the position that the ball strikes the floor (on the underside of the plane paper). Measure R , the horizontal range. This data allows you to determine the muzzle velocity by solving for it in Eq. (3) with $y = -H$ and $x = R$.
4. Now you are prepared to do the experiment. Do two runs. For two angles given to you by the instructor (say $\alpha_1 = 30^\circ$ and $\alpha_2 = 65^\circ$), calculate the trajectory from Eq. (3). by incrementing x from zero by 50 or 100 cm. Calculate $x = R$ for your measured $y = -H$ in each run and compare with your experimental values of R . Each run consists of 3 firings averaged.
5. Redo procedure 3. to be sure that the muzzle speed has not changed.