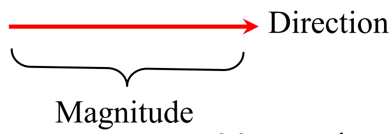
CHAPTER I
VECTORS, FORCES, EQULIBRIUM

AIM OF THE CHAPTER:
To learn about;
1. Vectors, scalars
2. Addition of vectors
   a) Geometric method
      - Parallelogram method
      - Head-to-tail
   b) Mathematical method.

KEYWORDS
Vector
Scalar
Equal vectors
Opposite vectors
Resultant vector
Parallelogram method
Tip-to-tail
Law of cosines
Law of sines

VECTORS AND SCALARS

A physical quantitiy such as , Velocity, or displacement, which has *direction* as well as *magnitude*, is called a **Vector**. Other quantities that are also vector are displacement, force, and mometum etc.



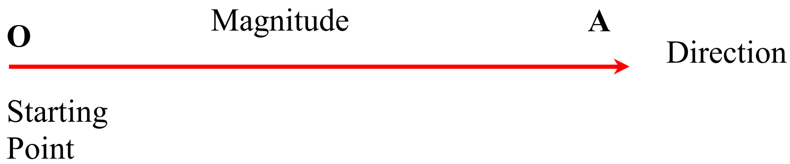
However, many quantities such as mass time and temperature have no direction associated with them. They are completely specified by giving numbers and units. Such quantities are called **Scalars**.

VECTOR → Magnitude + Direction → Velocity, Force and weight, Position, Acceleration, Moment , Impulse and ,Momentum, Displacement , Magnetic field Electric field, Gravity

SCALAR → Magnitude + No direction → Mass, Temperature, Volume Density, Work , Power, Energy, Heat, Time, Speed

When we write the symbol for a vector, we will always use **Boldface** type. For example , Velocity can be written as **V**. For a hand written books you can see the notation as $\vec{V}$ an arrow on the quantitiy.

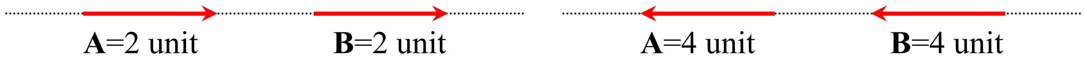
We have to have three properties to desciribe a vector.
1. Starting Point (Application Point)
2. Magnitude
3. Direction



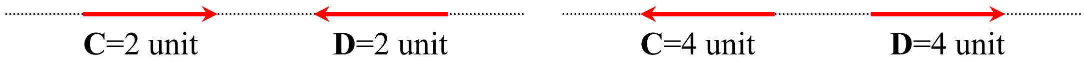
Magnitude of the vector, Sort of represents the strength of the vector. Bigger the arrow, Bigger the magnitude.

PROPERTIES OF VECTORS

Equal vectors: Two vectors with the same direction, and magnitude are called ***equal vevtors***. In the below given figure vectors a and b are equal vectors and **A = B**

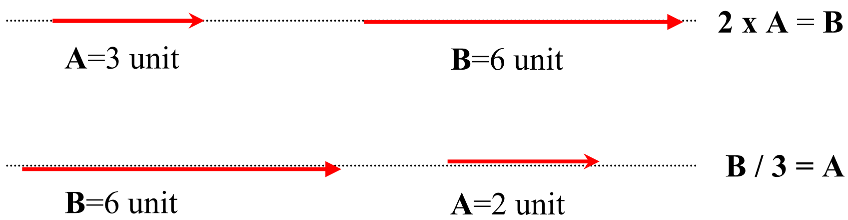


Opposite vectors: Two vectors with the same magnitude but in opposite direction are called ***opposite vectors***. In the figure vectors **C** and **D** are lie exactly in opposite directions so **C = -D**



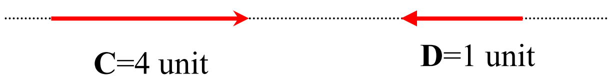
Multiplication / Division of vectors: If we multiply / Divide a vector by an integer number of n (n = 2,3,4,...etc.) again we obtain a new vector with a bigger / smaller magnitude in the same direction.

- If we multiply the **A** = 3 unit vector by 2 we end up with a new vector, **B** = 6 unit
- If we divide the **B** = 6 unit vector by 3 we end up with a new vector, **A** = 2 unit



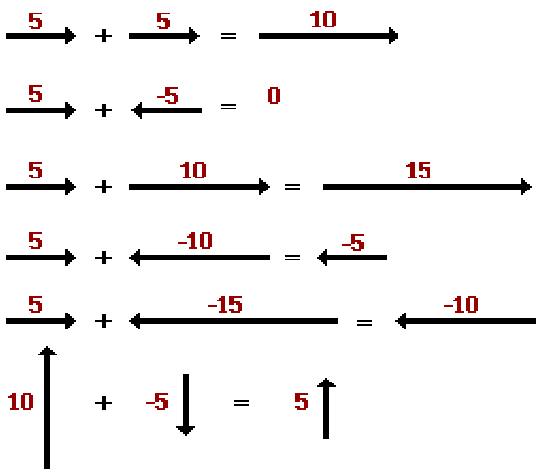
Resultant Vector: Result of two or more vector is called resultant vector.

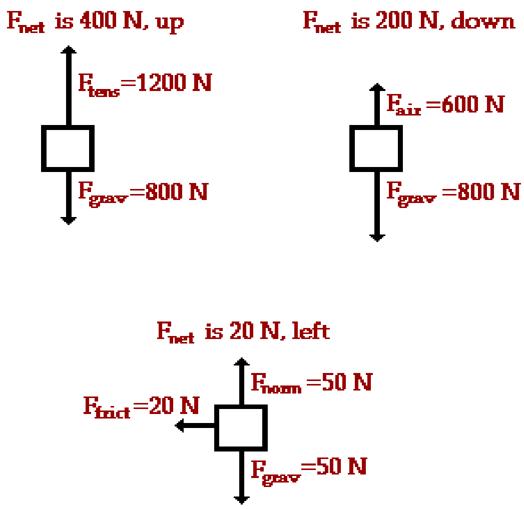
Subtraction of vectors: If we have two vectors with different magnitudes and in opposite directions, resultant vector can be found by mathematicaly subtracting these vectors.



Resultant vector of the system is **R = C – D** = 4-1 = 3 unit

EXAMPLE:





1) Vectors can be shifted in a plane without changing its magnitude , direction and line of action .

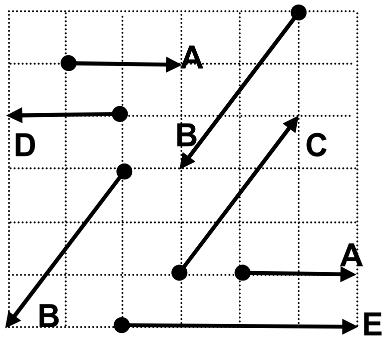


Fig.1.2.

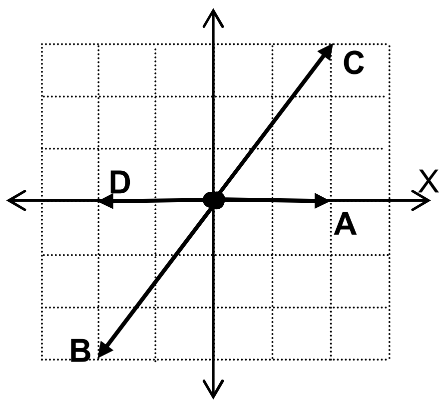


Fig.1.2.

2) Write examples for equal vectors by using the given vectors in Fig.1.2.1 . ...

3) Write two examples for equal vectors by using the given vectors in Fig.1.2.1. ....

4) If a vector quantity is multiplied (or divided) by a positive integer (n = 2 , 3, …) , we have a new vector having the same direction , line of action but different magnitude . Write two examples for equal vectors by using the given vectors in Fig.1.2.1. ....

5) If a vector quantity is multiplied (or divided) by a negative integer (n = -1 , -2 ,) , we have a new vector which is in the opposite direction of the original vector and has the same line of action but differentmagnitude . Write two examples for equal vectors by using the given vectors in Fig.1.2.1. ....

6) A vector in XY-plane can be resolved into its perpendicular components if its starting point coincide with the origin (0 ; 0) of the XY-coordinate system . Write the perpendicular components of the vectors **A** ; **B** , **C** and **D** , which are shown in Fig.1.2.2 , in the provded blanks below .

The horizontal component (x-component) of the vector **A** , $\mathbf{A_X}$ = …... unit
The vertical component (y-component) of the vector **A** , $\mathbf{A_y}$ = ... unit
The horizontal component (x-component) of the vector **B** , $\mathbf{B_X}$ = …... unit
The vertical component (y-component) of the vector **B** , $\mathbf{B_y}$ = ... unit
The horizontal component (x-component) of the vector **C** , $\mathbf{C_X}$ = …... unit
The vertical component (y-component) of the vector **C** , $\mathbf{C_y}$ = ... unit
The horizontal component (x-component) of the vector **D** , $\mathbf{D_X}$ = …... unit
The vertical component (y-component) of the vector **D** , $\mathbf{D_y}$ = ... unit

What should we do if we want to add two or more vector to each other?

ADDITION OF VECTORS

If we want to add two vectors to each other and find the resultant vector (R) there are two way of doing it so.

1. Geometric Method
   a. Parallelogram method
   b. Head –to-tail method
2. Mathematical Method
   a. Law of cosines
   b. Law of sines
   c. Resolving into components

Now let us try to investigate the properties of these methods.

1. GEOMETRİC METHOD : Suppose we have two vectors namely a and b. Both have different in direction and magnitude and we would like to add these two vector each other and find the resultant vector of these two:

a) **Parallelogram method:** The vectors are drawn from the same beginning point to form the adjacent sides of a parallelogram as shown in **fig 1-1a**. The parallelogram is then completed by drawing parallel lines to the two vectors **a** and **b**. The diagonal drawn from the beginning of the vector to the opposite corner of the parallelogram is the vector **c** representing the sum of **a** and **b fig 1-1b**.

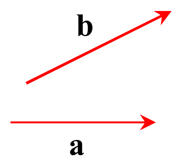


fig 1-1a

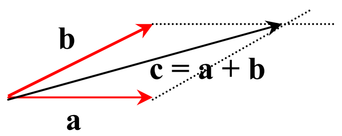
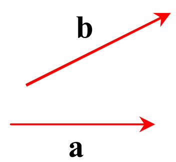


fig 1-1b

b) Head-to-tail: One of the vectors, **a** and **b,** is moved parallel to itself and they are drawn head-to-tail as shown in **Fig 1-2a-b.** Neither the direction nor the length of the vector is changed during this drawing. A third vector **c** is drawn from the tail of first vector to the head of the second vector. Vector **c** represents the sum of the resultant of vectors **a** and **b**.



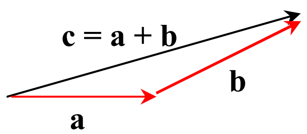


Fig.1-2a

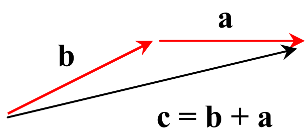
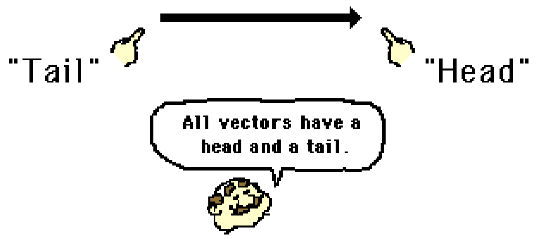


Fig.1-2b

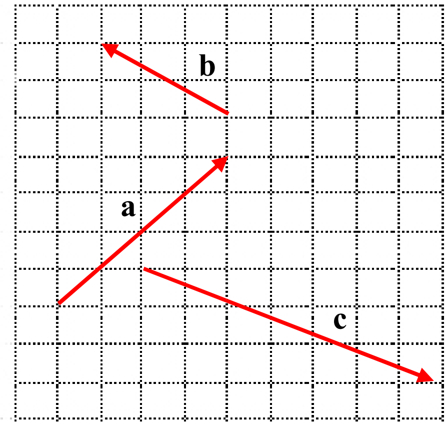


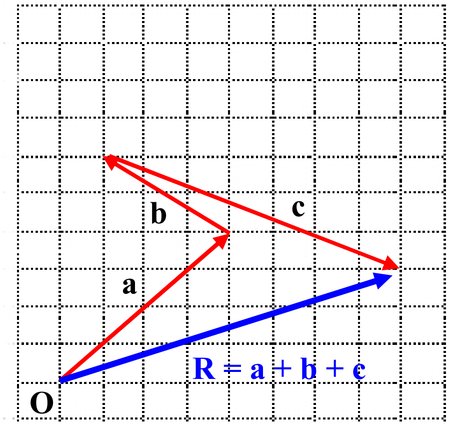
Animation

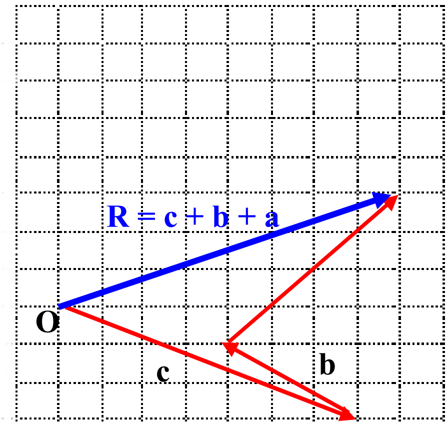
ADDITION OF SEVERAL VECTORS

We may often want to add three or more vectors. To determine the resultant of several vectors, follow the same procedure you use to add two vectors. Start from the any one of the given vectors, draw them head-to-tail one by one in the order you want. As an example consider the four vectors a, b, and c shown in the figure.

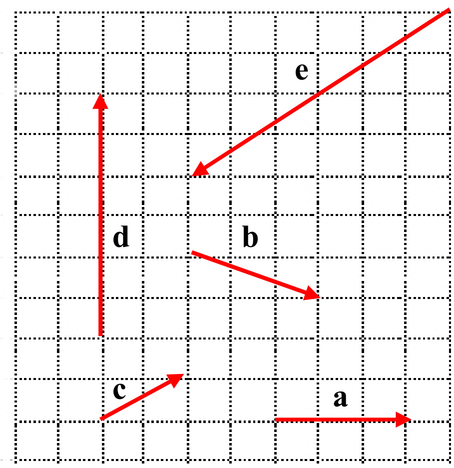
Calculate the resultant vector for the given vectors **a, b**, and **c**.

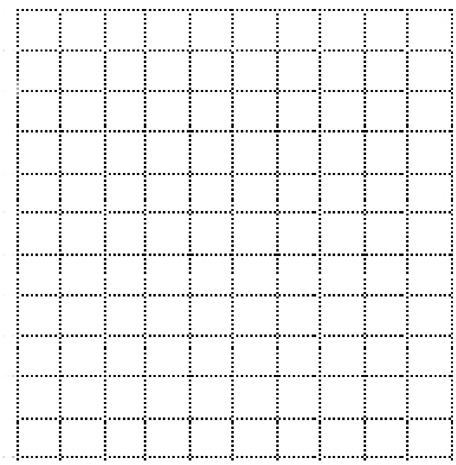




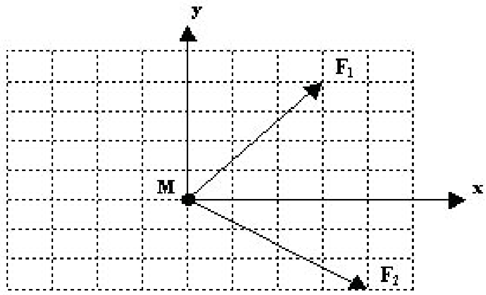


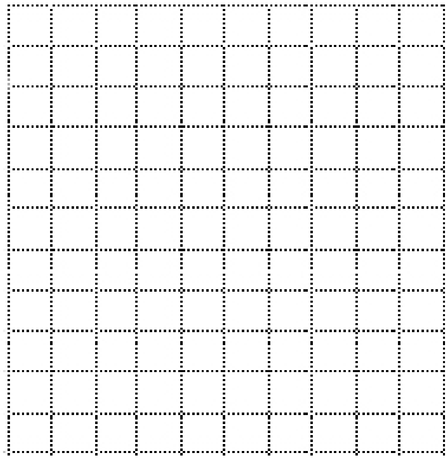
Calculate the resultant vector for the given vectors **a, b, c**, **d and e.**



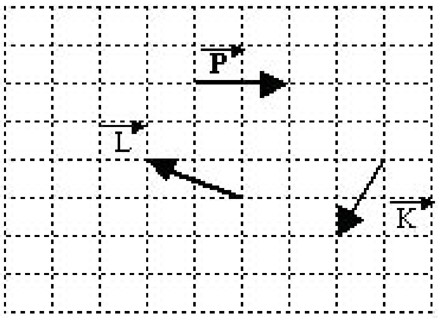


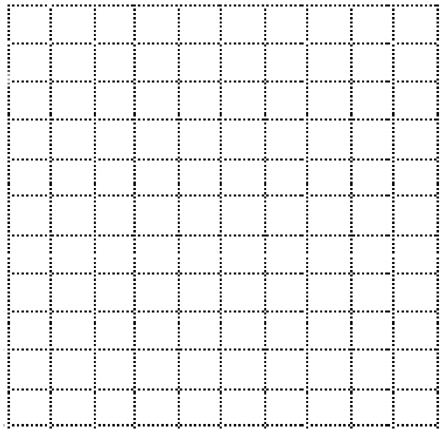
Three forces F_1, F_2, and F_3 are acting on the object M and object moves along the +x axis. If F_1 and F_2 are given in the figure find the magnitude of the third force F_3.





Vectors **P, K**, and **L** are given as shown in the figure. Calculate
K + 2P + L .

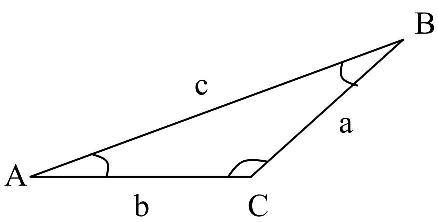




2- MATHEMATICAL METHOD:

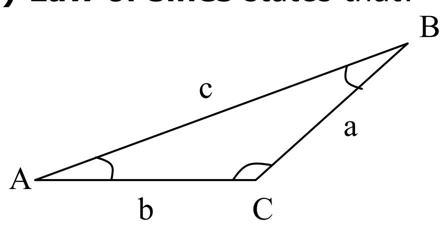
As we mentioned before another way of finding the resultant vector is to use a mathematical method. When two vectors are not perpendicular to each other, the magnitude and direction of their sum are defined by making use of the **law of cosines** , the **law of sines** or resolving into components.

a) Law of Cosines states that:



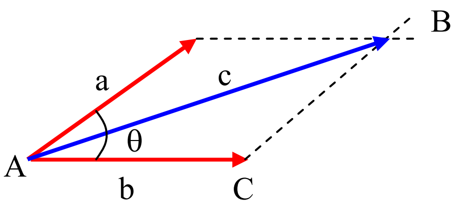
$$a^2 = b^2 + c^2 - 2\,b\,c\,\text{Cos}A$$
$$b^2 = a^2 + c^2 - 2\,a\,c\,\text{Cos}B$$
$$c^2 = a^2 + b^2 - 2\,a\,b\,\text{Cos}C$$

b) Law of sines states that:



$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

Or another useful mathematical method for finding the resultant vector is that :



$$c^2 = a^2 + b^2 + 2\,a\,b\,\text{Cos}\theta$$

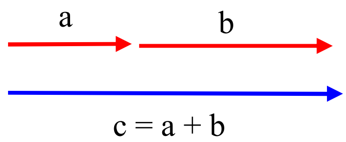
Special Cases :

1) If $\theta = 0$ then

Cos 0 = 1

$c^2 = a^2 + b^2 + 2ab = (a+b)^2$

$$c = a + b$$

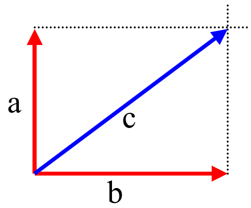


2) If $\theta = 90$ then

Cos 90 = 0

$c^2 = a^2 + b^2$

$$c = \sqrt{(a^2 + b^2)}$$

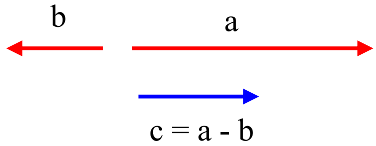


3) If $\theta = 180$ then

Cos 180 = -1

$c^2 = a^2 + b^2 - 2ab = (a - b)^2$

$$c = a - b$$



4) If $\theta = 60°$ and a = b then

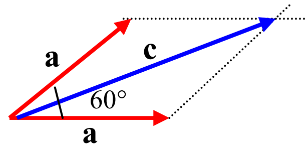
Cos 60 = 1 / 2

$c^2=a^2 + b^2 + 2\,ab\cos 60$ Since a=b

$c^2=a^2 + a^2 + 2\,aa\,½$

$c^2= 3\,a^2$

$$c = \sqrt{3}a$$



5) If $\theta = 90°$ and a = b then

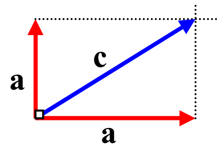
Cos 90 = 0

$c^2=a^2 + b^2 + 2\,ab\cos 90$ Since a=b

$c^2=a^2 + a^2$

$c^2= 2a^2$

$$c = \sqrt{2}a$$



6) If $\theta = 120°$ and a = b then

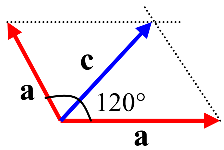
Cos 120 = -1 / 2

$c^2=a^2 + b^2 + 2\,ab\cos 120$ Since a=b

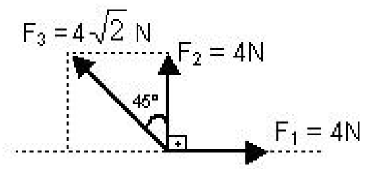
$c^2=a^2 + a^2 - 2\,aa\,1/2$

$c^2= a^2$

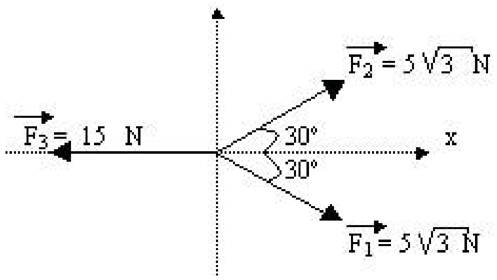
$$c = a$$



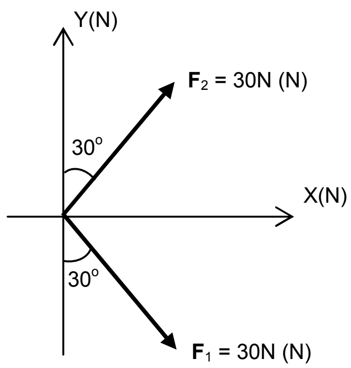
Find the resultant force / vector of the system.



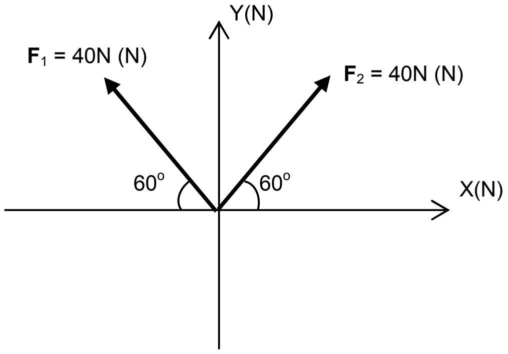
Find the resultant Vector / force of the given system.



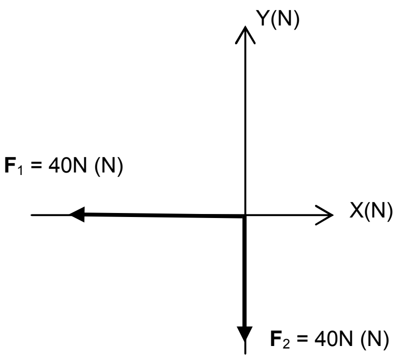
Find the magnitude and direction of the resultant force of the forces acting on a point object (o) given below.)



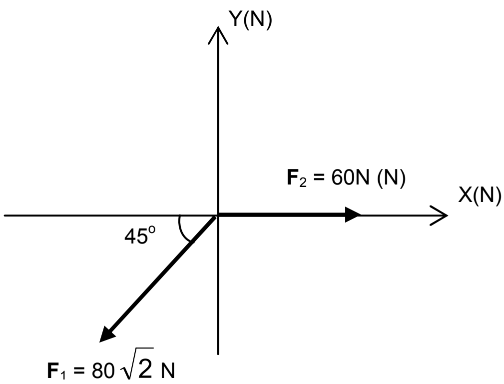
B)


Y(N)
F1 = 40N (N)
F2 = 40N (N)
60°
60°
X(N)

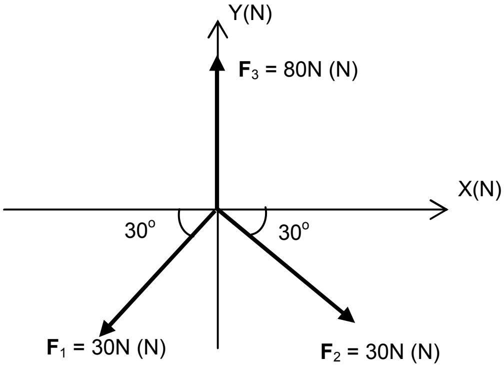
C)


Y(N)
F1 = 40N (N)
X(N)
F2 = 40N (N)

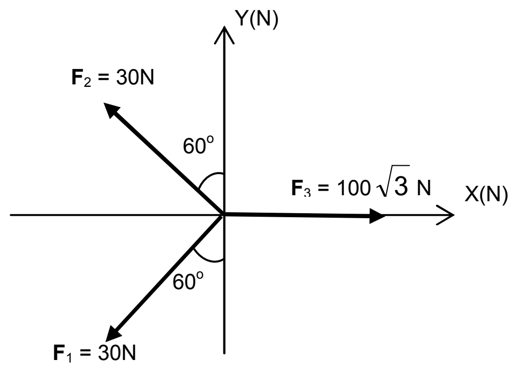
D)


Y(N)
F2 = 60N (N)
X(N)
45°
F1 = 80 √2 N

E)

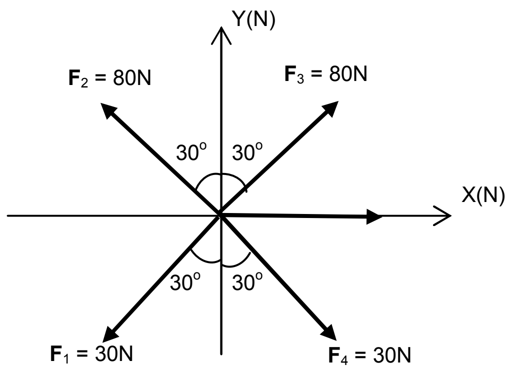

Y(N)
F3 = 80N (N)
X(N)
30°
30°
F1 = 30N (N)
F2 = 30N (N)

F)



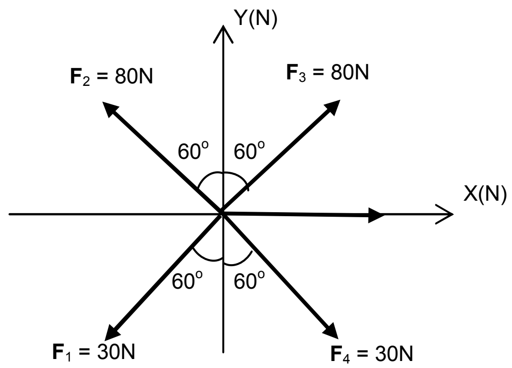
Y(N)
F₂ = 30N
60°
F₃ = 100 √3 N
X(N)
60°
F₁ = 30N

G)



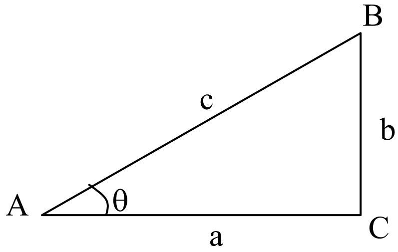
Y(N)
F₂ = 80N
F₃ = 80N
30°
30°
X(N)
30°
30°
F₁ = 30N
F₄ = 30N

H)



Y(N)
F₂ = 80N
F₃ = 80N
60°
60°
X(N)
60°
60°
F₁ = 30N
F₄ = 30N

LITTLE TRIGONOMETRI



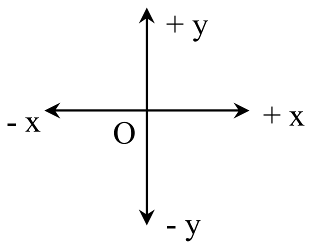
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{a}{c} \Rightarrow a = c\cos\theta$$

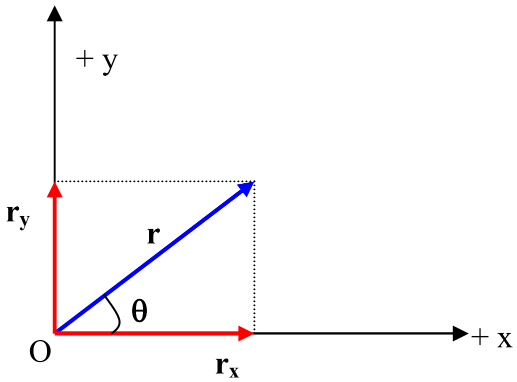
$$\sin\theta = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{b}{c} \Rightarrow b = c\sin\theta$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{b}{a} \Rightarrow b = a\tan\theta$$

$$c = \sqrt{a^2 + b^2}$$

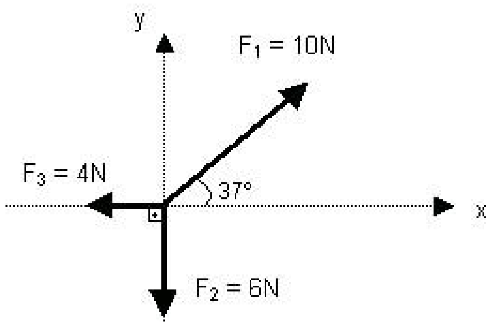
Resolving into components: It is always possible to resolve a vector into two components along any given pair of perpendicular directions. In order to resolve a vector into its own components we need a suitable cordinate system. We generally define the horizontal axis as x-axis and vertical axis as y-axis. Any given vector **r** can be drawn with the proper angle θ such that its tail coincides with the origin of the axes as shown. The components along the x- and y-axes of this vector is usually called $\mathbf{r_x}$ and $\mathbf{r_y}$ respectively**.** To find these components draw pependicular lines from the head of r to x and y axes. The magnitude of these components are then find using the suitable trigonometric equations.





$$r_x = r\cos\theta$$
$$r_y = r\sin\theta$$
$$r = \sqrt{r_x^2 + r_y^2}$$
$$\tan\theta = \frac{r_y}{r_x}$$

Find the resultant force of the given system. (Sin37 = 0.6 , Cos37 = 0.8)



Assume that you are given two vectors **A** and **B** as shown in Figure

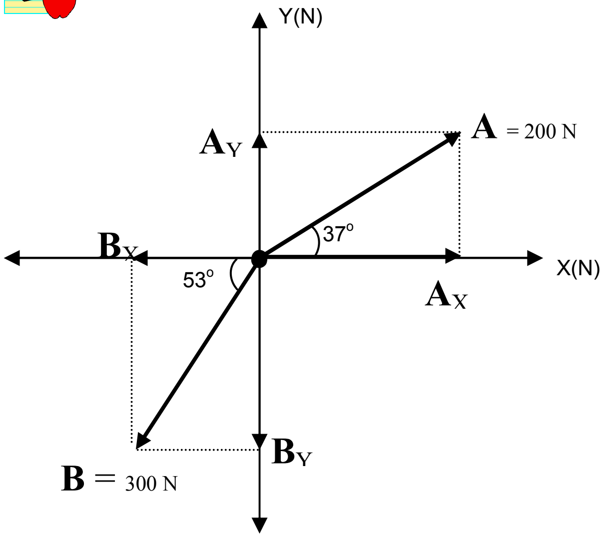


Fig.

The magnitudes of the perpendicular components , $\mathbf{A_X}$ and $\mathbf{A_Y}$, of the vector **A** are ;

$A_X = A \,.\, Cos37^o$ $A_Y = A \,.\, Sin37^o$

= ……………. =……………

We may calculate the magnitude of the vector **A** by using its perpendicular components $\mathbf{A_X}$ and $\mathbf{A_Y}$ by applying Pythagorian theory . ;

$A^2 = (A_X)^2 + (A_Y)^2$

= …………………..

= …………………..

The magnitudes of the perpendicular components , $\mathbf{B_X}$ and $\mathbf{B_Y}$, of the vector **B** are ;

$B_X = B \,.\, Cos53^o$ $B_Y = B \,.\, Sin53^o$

= ……………. = ……………

= …………….. = ……………

We may calculate the magnitude of the vector **B** by using its perpendicular components $\mathbf{B_X}$ and $\mathbf{B_Y}$ by applying Pythagorian theory . ;

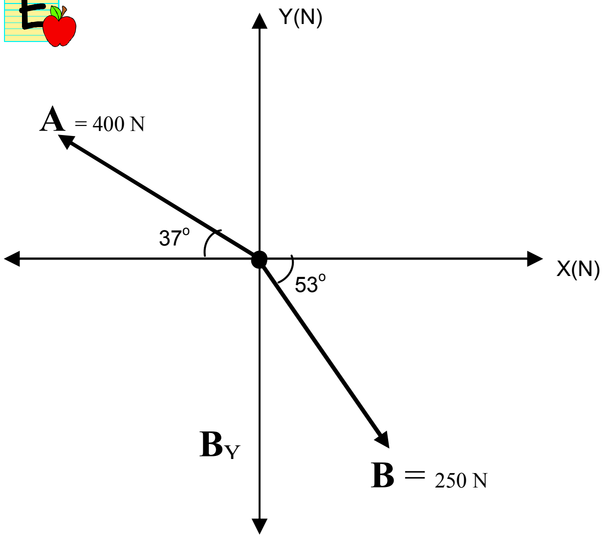
$B^2 = (B_X)^2 + (B_Y)^2$

= …………………..

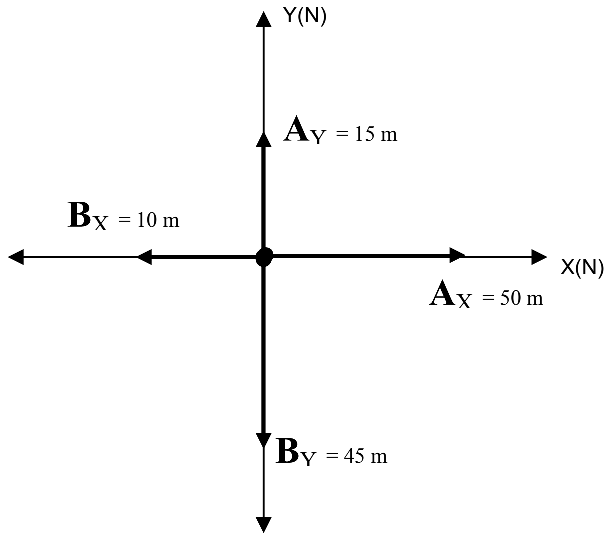
= …………………..

= …………………

Assume that you are given two vectors **A** and **B** as shown in Figure Calculate and draw the magnitudes of the perpendicular components of the vectors **A** and **B** .



Assume that you are given the perpendicular components of the vectors **A** and **B** as shown in Fig. 1.2.7 .

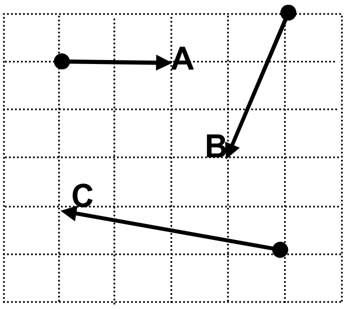


a) Calculate the magnitudes of the vectors **A** and **B** .

b) Draw the vectors **A** and **B** .

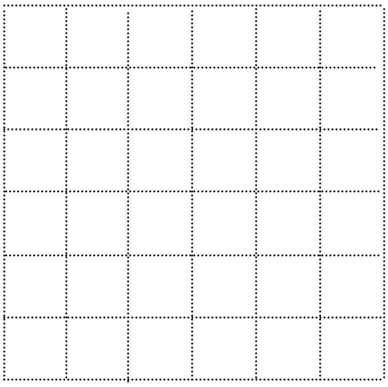
c) What are the directions of the vectors **A** and **B** ? (Hint : Direction of a vector is given by an angle between the vector and positive x-axis . You need a trigonometric table , too !)

Mathematical Method



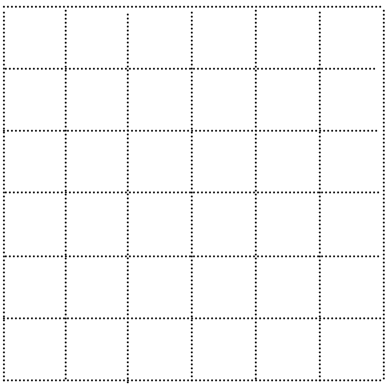
Let’s assume that a resultant vector **R** is defined as **R** = **A** + **B** + **C** , and the vectors **A** , **B** and **C** are as shown in Figure .

<u>Component Method</u>



Find the magnitudes of the perpendicular components of the vectors **A** , **B** and **C** shown in Figure , and record them into the table near the grid in Fig.14 . Add up the magnitudes of the x-components to find the magnitude of the x-component ($\mathbf{R}_x$) of the resultant vector **R** . Add up the magnitudes of the y-components to find the magnitude of the y-component ($\mathbf{R}_y$) of the resultant vector **R** . Then, use these components to draw the resultant vector **R** in
Figure

	X-component	Y-component
A		
B		
C		
R		



Let’s assume that a resultant vector **R** is defined as **R** = **A** + **B** - **C** , and the vectors **A** , **B** and **C** are as shown in Figure . Draw the resultant vector
R = **A** + **B** – **C** in Figure, calculate its magnitude and find its direction .

	X-component	Y-component
A		
B		
C		
R		

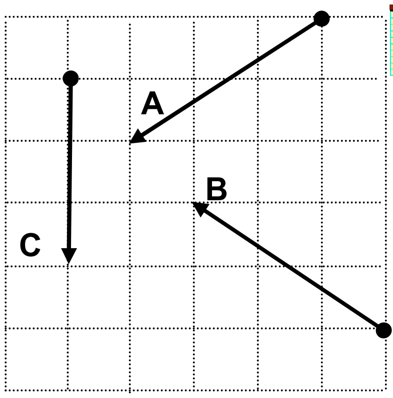
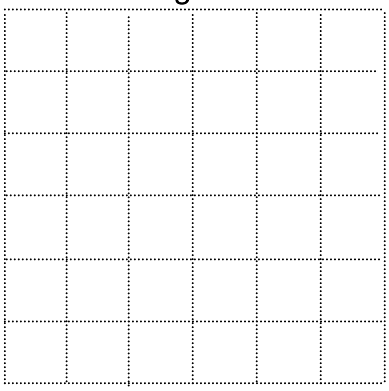
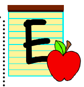


Fig.





Assume that you are given three vectors **A** , **B** and **C** as shown in Figure and a resultant vector **R** is defined as **R** = **A** + **B** - **C** .

a) Draw the resultant vector **R** on the provided grid below Figure by using
(component) method .

b) Calculate the magnitude of the resultant vector **R** .

c) Find the direction of the resultant vector **R** .

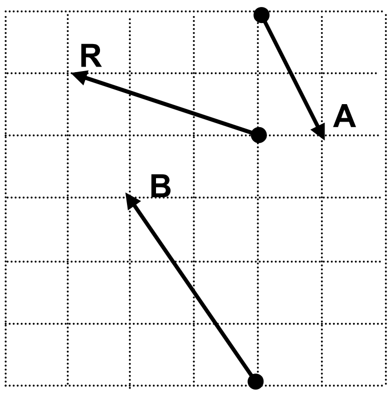
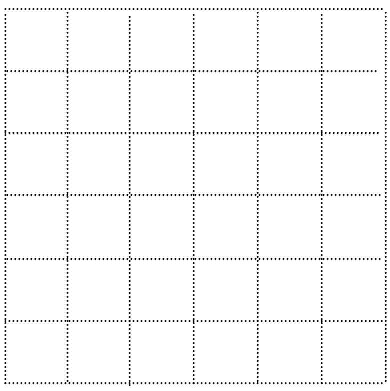


Fig.



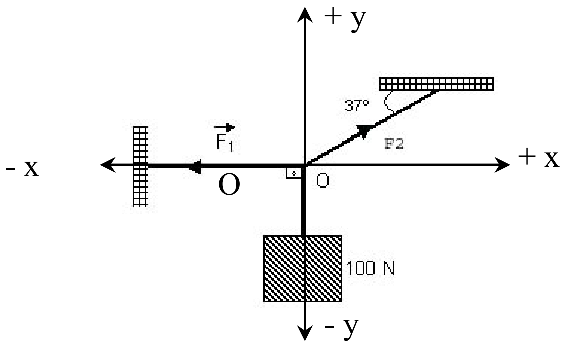
Assume that you are given three vectors **A** and **B** as shown in
Figure , and a resultant vector **R** which is defined as **R** = **A** + **B** - **C** .

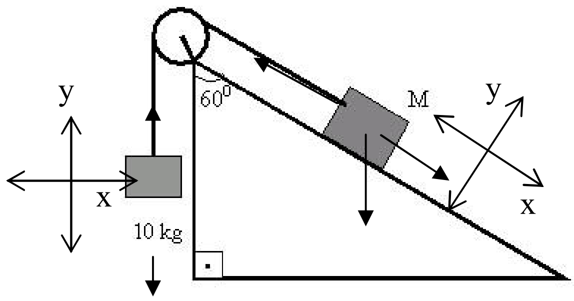
a) Find the vector **C** on the provided grid below Figure by using component
method .

b) Calculate the magnitude of the vector **C** .

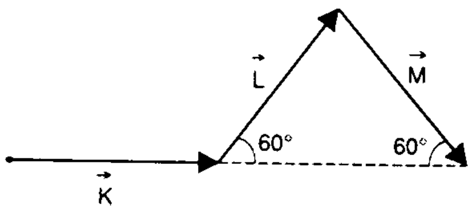
c) Find the direction of the vector **C** .

This coordinate system may change its position i.e. rotate, depending on the forces acting on the system





Q.9)



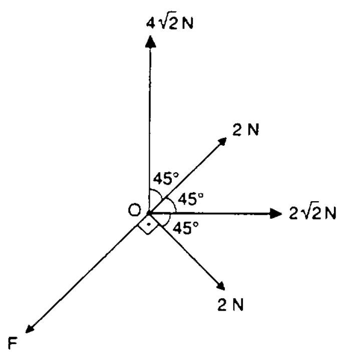
The vectors given in the figure above are on the same plane and equal in magnitude.

If **R** = **K** -2 (**L** + **M**) , the vector **R** is equal to;

a) 2**K** b) – **K** c) **K**

d) – 2**K** e) 3**K**

Q.10)

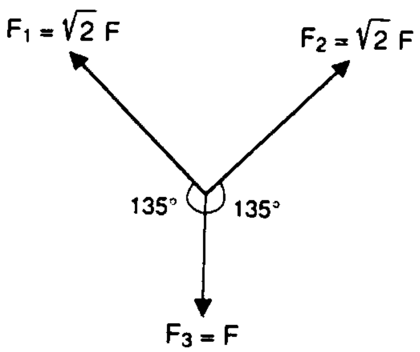


The point object at O is in equilibrium under the action of coplanar forces shown in the figure.

What is the magnitude of **F** .

a) 2 b) 4 c) 8 d) 16 e) 32

Q.12)



When $\mathbf{R}_1 = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$, the magnitude of $\mathbf{R}_1$ is R.

If $\mathbf{R}_2 = \mathbf{F}_1 + \mathbf{F}_2 - \mathbf{F}_3$, what is the magnitude of $\mathbf{R}_2$ in terms of R. ?

a) F b) $\sqrt{2}$ F c) $\sqrt{3}$ F d) 2F e) 3F