



## Newton's law of Motion



### AIM OF THE CHAPTER:

- to understand Newton's law of motions
- To identify the action reaction pairs.
- Define inertia.
- Define friction.
- To understand air friction



### KEYWORDS

Inertia  
State of motion  
Action-reaction  
Tension  
Friction  
Static friction  
Kinetic friction  
Balance

### *The Meaning of Force*

A force is a push or pull upon an object resulting from the object's interaction with another object. Whenever there is an interaction between two objects, there is force acting on each of the objects. When the interaction ceases, the two objects no longer experience a force. Forces only exist as a result of an interaction.

For simplicity sake, all forces (interactions) between objects can be placed into two broad categories:

- contact forces, and
- forces resulting from action-at-a-distance

**Contact forces** are types of forces in which the two interacting objects are physically in contact with each other.

**Action-at-a-distance forces** are types of forces in which the two interacting objects are not in physical contact with each other, but are able to exert a push or pull despite the physical separation.

#### **Contact Forces**

Frictional Force

Tensional Force

Normal Force

Air Resistance Force

Applied Force

Spring Force

#### **Action-at-a-Distance Forces**

Gravitational Force

Electrical Force

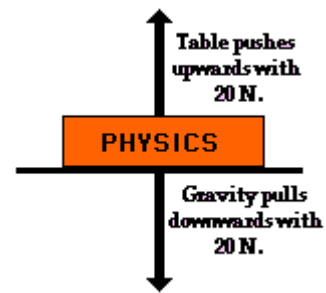
Magnetic Force

Force is a quantity which is measured using a standard metric unit known as the Newton. One Newton is the amount of force required to give a 1-kg mass an acceleration of 1 m/s<sup>2</sup>. A Newton is abbreviated by an "N." If you say "10.0 N," you mean 10.0 Newtons of force. Thus, the following unit equivalency can be stated:

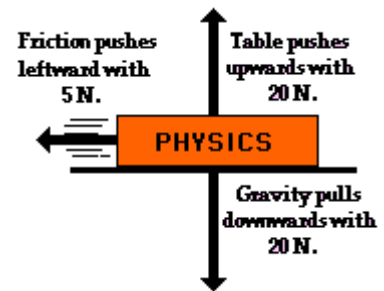
$$1 \text{ Newton} = 1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}$$



Force is a **vector quantity**. a vector quantity is a quantity which has both **magnitude** and **direction**. To fully describe the force acting upon an object, you must describe both its magnitude (size) and its direction. Thus, 10 Newtons is not a full description of the force acting upon an object. In contrast, 10 Newtons, downwards is a complete description of the force acting upon an object; both the magnitude (10 Newtons) and the direction (downwards) are given.



Because force is a vector and has direction, it is common to represent forces using diagrams in which the force is represented by an **arrow**. Such vector diagrams were introduced earlier and will be used throughout your study of physics. The size of the arrow is reflective of the magnitude of the force and the direction of the arrow reveals the direction in which the force is acting. Furthermore, because forces are vectors, the influence of one individual force upon an object is often canceled by the influence of another force acting on the same object. For example, the influence of a 20-Newton upward force acting upon a book is canceled by the influence of a 20-Newton downward force acting upon the book. In such instances, the two individual forces are said to "**balance each other**".



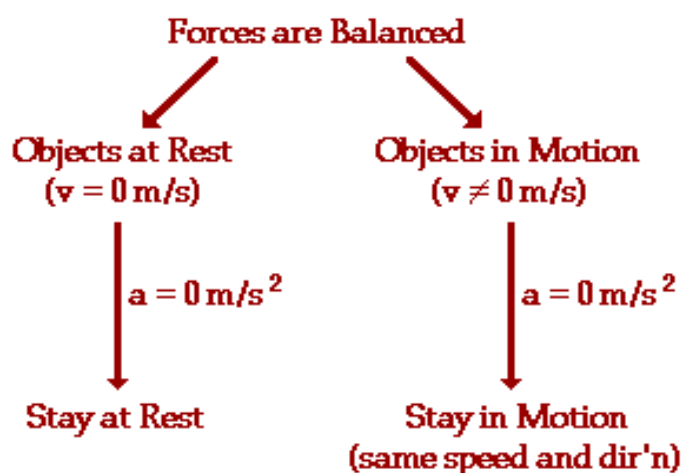
Other situations could be imagined in which two of the individual vector forces cancel each other ("**balance**") and a third individual force exists that is not balanced by another force. For example, imagine a book sliding across the rough surface of a table from left to right. The downward force of gravity and the upward force of the table supporting the book are of equal magnitude, act in opposite directions and thus balance each other. However, the force of friction acts leftwards, and there is no rightward force to balance it. In this case, an unbalanced force acts upon the book to change its **state of motion** and the book slows down.



## Newton's First Law

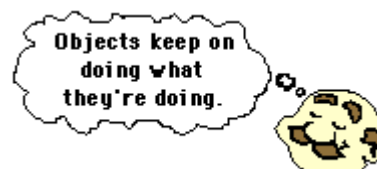
In a previous unit of study, the variety of ways by which motion can be described (words, graphs, diagrams, numbers, etc.) was discussed. In this unit (Newton's Laws of Motion), the ways in which motion can be explained will be discussed. **Isaac Newton** (a 17th century scientist) put forth a variety of laws which explain why objects move (or don't move) as they do. These three laws have become known as Newton's three laws of motion. The focus of this lesson is Newton's first law of motion - sometimes referred to as the "law of inertia."

Newton's first law of motion is often stated as "An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force."



There are two parts to this statement - one which predicts the behavior of stationary objects and the other which predicts the behavior of moving objects. The two parts are summarized in the following diagram.

The behavior of all objects can be described by saying that objects tend to "**keep on doing what they're doing**" (unless acted upon by an unbalanced force). If at rest, they will continue in this same state of rest. If in motion with an eastward velocity of 5 m/s, they will continue in this same state of motion (5 m/s, East). If in motion with a leftward velocity of 2 m/s, they will continue in this same state of motion (2 m/s, left). The state of motion of an object is maintained as long as the object is not acted upon by an unbalanced force. All objects resist changes in their state of motion - they tend to "**keep on doing what they're doing**."



There are many applications of Newton's first law of motion. Consider some of your experiences in an automobile. Have you ever observed the behavior of coffee in a coffee cup filled to the rim while starting a car from rest or while bringing a car to rest from a state of motion? Coffee tends to "keep on doing what it is doing." When you accelerate a car from rest, the road provides an unbalanced force on the spinning wheels to push the car forward; yet the coffee (which is at rest) wants to stay at rest. While the car accelerates forward, the coffee remains in the same position; subsequently, the car accelerates out from under the coffee and the coffee spills in your lap. On the other hand, when braking from a state of motion the coffee continues to move forward with the same speed and in the same direction, ultimately hitting the windshield or the dashboard. Coffee in motion tends to stay in motion.



There are many more applications of Newton's first law of motion. Several applications are listed below - it is hoped that you could provide explanations for each application.

- the head of a hammer can be tightened onto the wooden handle by banging the bottom of the handle against a hard surface.
- headrests are placed in cars to prevent whiplash injuries during rear-end collisions.
- while riding a skateboard (or wagon or bicycle), you fly forward off the board when hitting a curb or rock or other object which abruptly halts the motion of the skateboard.



### **Inertia, State of Motion (V, a)**

Inertia is the tendency of an object to resist changes in its state of motion. But what is meant by the phrase "state of motion?" The state of motion of an object is defined by its velocity - the speed with a direction. Thus, inertia could be redefined as follows:

**Inertia = tendency of an object to resist changes in its velocity.**  
**Inertia = tendency of an object to resist changes in its acceleration.**

The law of inertia is most commonly experienced when riding in cars and trucks. In fact, the tendency of moving objects to continue in motion is a common cause of a variety of transportation accidents - of both small and large magnitudes. Consider for instance the unfortunate collision of a car with a wall. Upon contact with the wall, an unbalanced force acts upon the car to abruptly decelerate it to rest. Any passengers in the car will also be decelerated to rest if they are strapped to the car by seat belts. Being strapped tightly to the car, the passengers share the same state of motion as the car. As the car accelerates, the passengers accelerate with it; as the car decelerates, the passengers decelerate with it; and as the car maintains a constant speed, the passengers maintain a constant speed as well.

But what would happen if the passengers were not wearing the seat belt? What motion would the passengers undergo if they failed to use their seat belts and the car were brought to a sudden and abrupt halt by a collision with a wall? Were this scenario to occur, the passengers would no longer share the same state of motion as the car. The presence of the strap assures that the forces necessary for accelerated and decelerated motion exist. Yet, once the strap is no longer present to do its *job*, the passengers are more likely to maintain its state of motion.

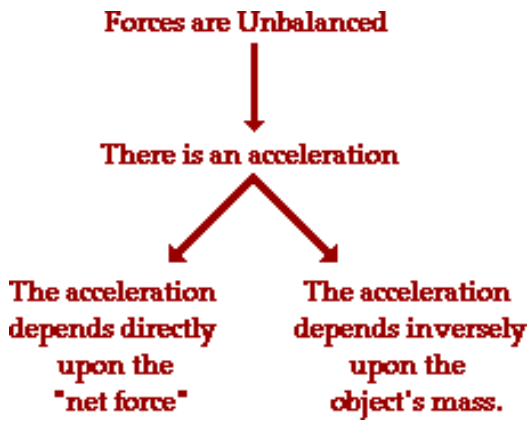


If the car were to abruptly stop and the seat belts were not being worn, then the passengers in motion would continue in motion. Assuming a negligible amount of friction between the passengers and the seats, the passengers would likely be propelled from the car and be hurled into the air. Once they leave the car, the passengers becomes projectiles and continue in projectile-like motion.



# Newton's Second Law of Motion

## Newton's Second Law

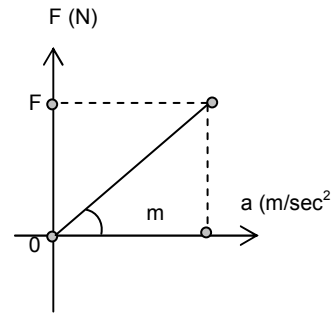


Newton's second law of motion pertains to the behavior of objects for which all existing forces are not balanced. *The second law states that the acceleration of an object is dependent upon two variables - the net force acting upon the object and the mass of the object.*

Newton's second law of motion can be formally stated as follows:

The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object. In terms of an equation, the net force is equated to the product of the mass times the acceleration.

$F_{net} = m \times a$	<b>Newton' second law</b>
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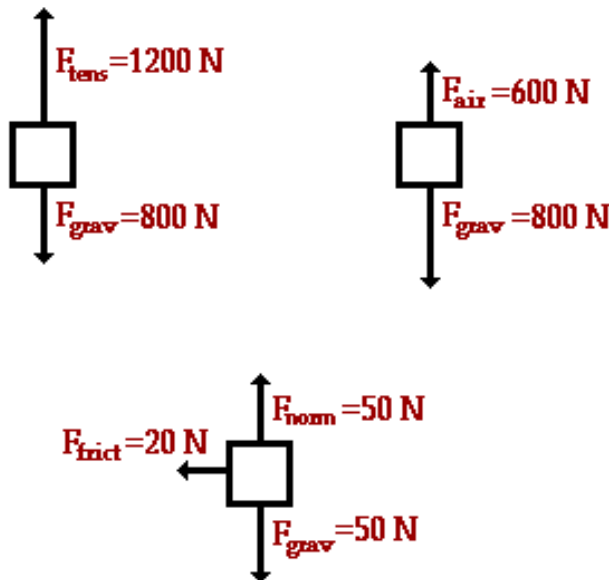


In this entire discussion, the emphasis has been on the "net force." The acceleration is directly proportional to the "net force;" the "net force" equals mass times acceleration; the acceleration in the same direction as the "net force;" an acceleration is produced by a "net force." The NET FORCE. It is important to remember this distinction. Do not use the value of merely "any force" in the above equation; it is the net force which is related to acceleration. The net force is the vector sum of all the forces. If all the individual forces acting upon an object are known, then the net force can be determined.

The above equation also indicates that a unit of force is equal to a unit of mass times a unit of acceleration. By substituting standard metric units for force, mass, and acceleration into the above equation, the following unit equivalency can be written.

$$1 \text{ Newton} = 1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}$$

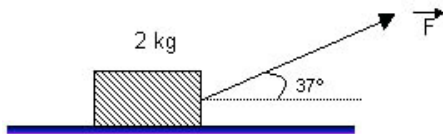
The definition of the standard metric unit of force is stated by the above equation. One Newton is defined as the amount of force required to give a 1-kg mass an acceleration of 1 m/s<sup>2</sup>



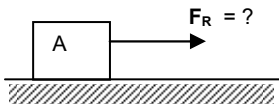


**Example 1** A 10-kg body has an acceleration of  $5 \text{ m/s}^2$ . Find the net force acting on the object.

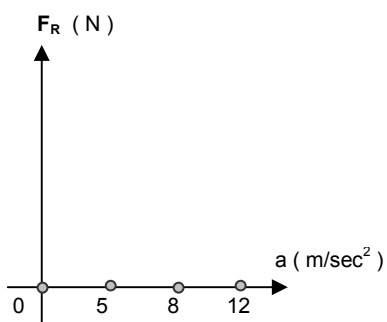
**Example 2** If a force of  $F=10 \text{ N}$  is acting on a  $2 \text{ kg}$  object; find the acceleration of the object.  
( $\sin 37^\circ = 0.6$ ,  $\cos 37^\circ = 0.8$ )



**Example 3**



A  $4 \text{ kg}$  box is horizontally pulled on a straight frictionless road as shown in the figure. What should the magnitude of the resultant force if the acceleration is ;



a)  $5 \text{ m/sec}^2$

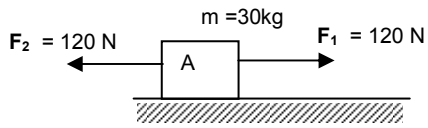
b)  $8 \text{ m/sec}^2$

c)  $12 \text{ m/sec}^2$

d) Draw its F-a graph.



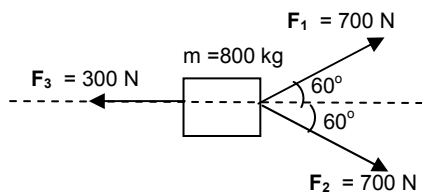
#### Example 4



An object of 30 kg is horizontally pulled on a straight, horizontal and frictionless road as shown in the figure.

- What is the net force acting on the object?
- What is the acceleration of the object?
- In which direction does the object move? Explain why?

#### Example 5

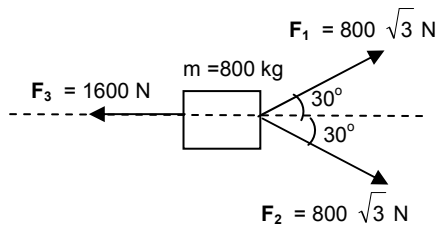


800 kg car is pulled by three forces on a horizontal and frictionless road as shown in the figure.

- What is the net force acting on the car ?
- What is the acceleration of the car ?
- In which direction does the car move? Explain why ?



**Example 6 (Homework)**



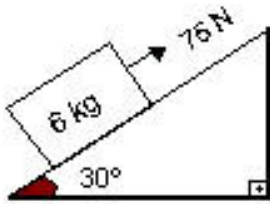
A car of 800 kg is horizontally pulled on a straight, horizontal and frictionless road as shown in the figure. (Assume  $\cos 30 = \frac{\sqrt{3}}{2}$ ,  $\sin 30 = \frac{1}{2}$ )

a) What are the magnitude and direction of the resultant force acting on the object ?  
**(Ans:  $F_{\text{Net}} = 800\text{N}$ )**

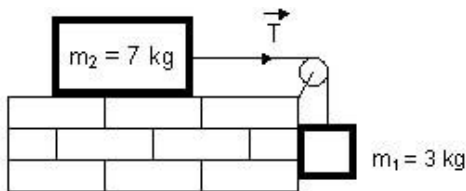
b) What is the acceleration of the object? **(Ans:  $a = 1 \text{ m/s}^2$ )**

c) In which direction does the object move? Explain why? **(Ans: +x)**

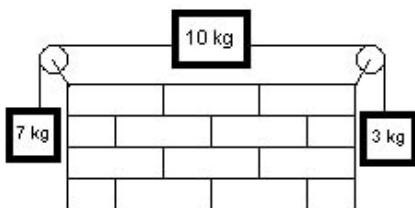
**Example 7** A 6-kg object is pulled with a force of 75 N along the frictionless inclined plane. Find the acceleration of the block. ( $\sin 30 = 0.5$ ,  $\cos 30 = 0.8$ )



**Example 8** Find the tension on the string if the whole system is frictionless. ( $g = 10 \text{ m/s}^2$ )



**Example 10** Find the acceleration of the given system if the system is frictionless. ( $g = 10 \text{ m/s}^2$ )





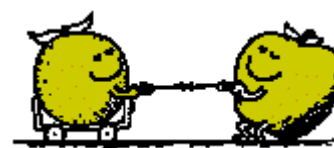
## Newton's Third Law of Motion

### Newton's Third Law

A force is a push or a pull upon an object which results from its interaction with another object. Forces result from interactions! Some forces result from *contact interactions* (normal, frictional, tensional, and applied forces are examples of contact forces) and other forces are the result of action-at-a-distance interactions (gravitational, electrical, and magnetic forces). According to Newton, whenever objects A and B interact with each other, they exert forces upon each other. When you sit in your chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body. There are two forces resulting from this interaction - a force on the chair and a force on your body. These two forces are called *action* and *reaction* forces and are the subject of Newton's third law of motion. Formally stated, Newton's third law is:

*"For every action, there is an equal and opposite reaction."*

The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object.



Forces always come in pairs - equal and opposite action-reaction force pairs.

A variety of action-reaction force pairs are evident in nature. Consider the propulsion of a fish through the water. A fish uses its fins to push water backwards. But a push on the water will only serve to accelerate the water. In turn, the water *reacts* by pushing the fish forwards, propelling the fish through the water. The size of the force on the water equals the size of the force on the fish; the direction of the force on the water (backwards) is opposite the direction of the force on the fish (forwards). For every action, there is an equal (in size) and opposite (in direction) reaction force. Action-reaction force pairs make it possible for fish to swim.

Consider the flying motion of birds. A bird flies by use of its wings. The wings of a bird push air downwards. In turn, the air reacts by pushing the bird upwards. The size of the force on the air equals the size of the force on the bird; the direction of the force on the air (downwards) is opposite the direction of the force on the bird (upwards). For every action, there is an equal (in size) and opposite (in direction) reaction. Action-reaction force pairs make it possible for birds to fly.

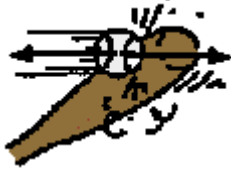


Consider the motion of your automobile to school. An automobile is equipped with wheels which spin backwards. As the wheels spin backwards, they push the road backwards. In turn, the road reacts by pushing the wheels forward. The size of the force on the road equals the size of the force on the wheels (or automobile); the direction of the force on the road (downwards) is opposite the direction of the force on the wheels (upwards). For every action, there is an equal (in size) and opposite (in direction) reaction. Action-reaction force pairs make it possible for automobiles to move.

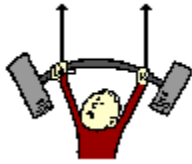


### Identifying Action and Reaction Force Pairs

According to Newton's third law, for every action force there is an equal (in size) and opposite (in direction) reaction force. Forces always come in pairs - known as "action-reaction force pairs." Identifying and describing action-reaction force pairs is a simple matter of identifying the two interacting objects and making two statements describing *who is pushing on who* and in what direction. For example, consider the interaction between a baseball bat and a baseball.



The baseball forces the bat to the right (an action); the bat forces the ball to the left (the reaction). Note that the nouns in the sentence describing the action force switch places when describing the reaction force.



Athlete pushes bar upwards.

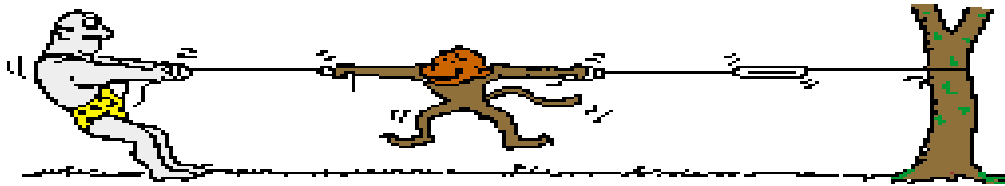


Bowling ball pushes pin rightwards



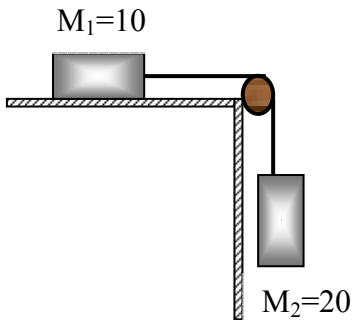
Balloon wall pushes compressed air inwards.

Identify at least six pairs of action-reaction force pairs in the following diagram.

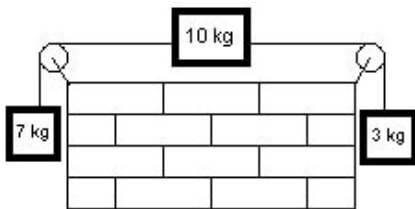




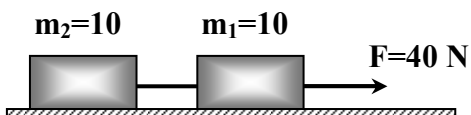
**Example 9** Find out the acceleration and the tension between the blocks. ( $g=10\text{m/s}^2$ )



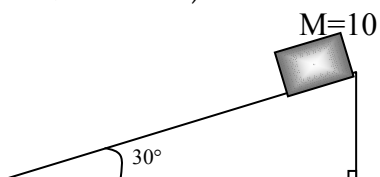
**Example 10** Find the acceleration of the given system. ( $g =10 \text{ m/s}^2$ )



**Example 11** two boxes with a mass of  $m_1=10 \text{ kg}$  and  $m_2= 10 \text{ kg}$  are connected together with a light string and a force of  $40 \text{ N}$  applied to the system. (a) Find the acceleration of each block. (b) Find the tension between the blocks.



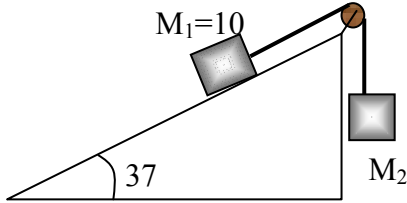
**Example 12** A  $10 \text{ kg}$  object is released from the top of the inclined plane. (a) find the normal force acting on the object. (b) find the acceleration of the block. ( $\text{Cos}30=0.8$ ,  $\text{sin}30=0.5$ )



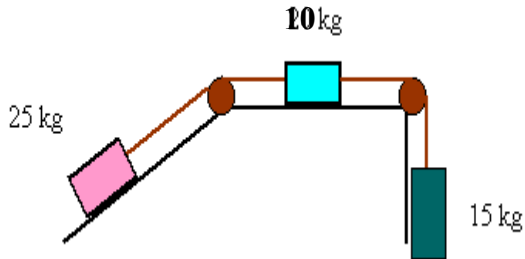


**Example 11** A block of mass  $m_1=10$  kg on a smooth inclined plane of angle of  $37^\circ$  is connected by a string over a pulley to a second mass of  $m_2$  hanging vertically. If the acceleration of the mass 2 is  $2.5$  m/s<sup>2</sup> downward then find: (Sin $37=0.6$ , cos $37=0.8$ )

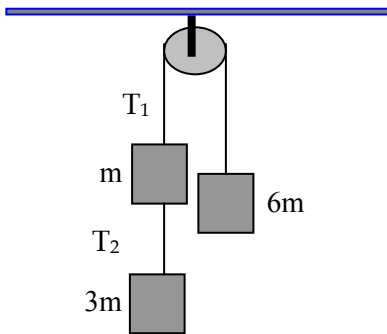
- a) reaction force of the inclined plane.
- b) Tension in the string
- c) Magnitude of the mass  $m_2$



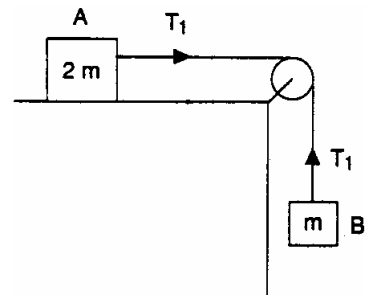
**Example 12** Take the three masses and two cords as shown below. Find the acceleration of the system and the tension in the cords. (Angle of the incline plane is  $30^\circ$ )



**Example 13** If the system is released calculate the ratio of  $T_1 / T_2 = ?$



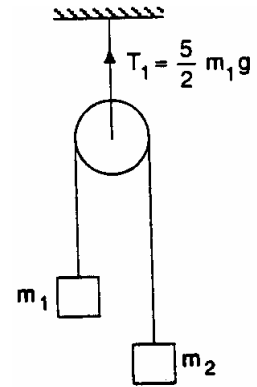
**Example 14:** The system in the figure above is frictionless. The tension in the rope connecting the objects A and B is  $T_1$  . When the objects are replaced , the tension in the same rope becomes  $T_2$  . What is the ratio  $T_1 / T_2$  ?





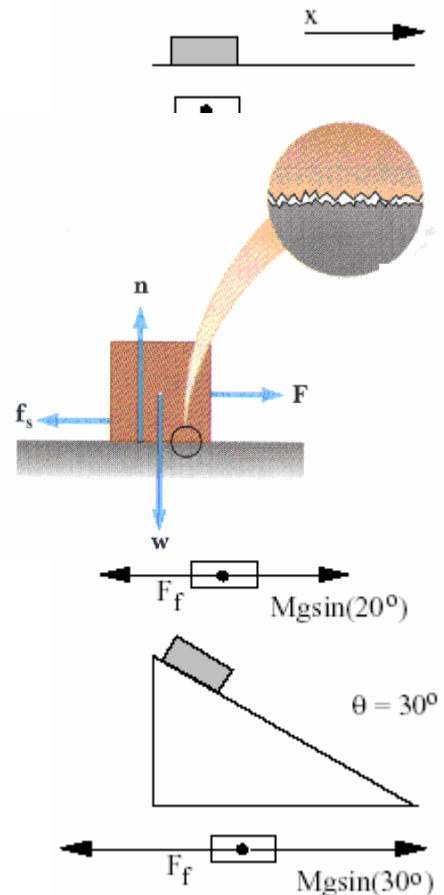
**Example 15**

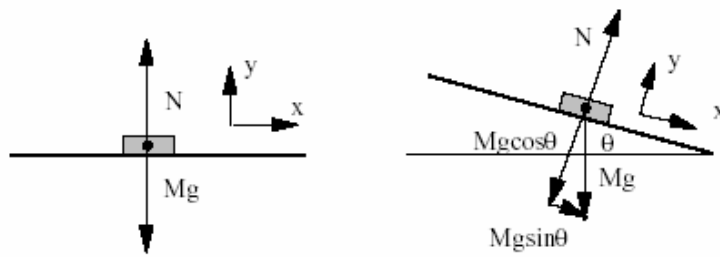
The tension in the rope connecting the frictionless fixed pulley to the ceiling is  $T_1 = (5/2)m_1 g$ . What is the ratio  $m_1 / m_2$ ?



**Friction**

Let us begin with an observation. We place an object with a mass  $M$  on a horizontal surface. Since the object doesn't move, the sum of the forces in the  $x$  and  $y$  directions must be zero. There are no forces on the object in the horizontal direction. In the  $y$ -direction we have  $N - Mg = 0$ , that is the normal force is equal and opposite to the weight of the object. We now start to tip the surface at a small angle  $\theta$ . We keep our  $x$ -direction in the direction of the planar surface. Again, we write down the forces in the  $x$  and  $y$  directions





$$\sum F_y = N - Mg \cos \theta = 0$$

$$\sum F_x = Mg \sin \theta \neq 0$$

## Static Friction

The equation for  $F_x$  says that the force is nonzero so we should immediately observe the object to accelerate down the plane, no matter how small the angle  $\theta$ .

This is almost never observed. What we observe is that the object remains stationary on the plane. Therefore, there must be a force in the x-direction that we have not considered. This force is the force of **friction**. Our force equation in the x direction becomes

$$F_x = Mg \sin \theta - F_f,$$

where  $F_f$  is the force of friction.

Now, what happens as we make the angle steeper?

By increasing the angle, we increase the force in the x-direction. (Remember, the force in the x-direction is  $Mg \sin \theta$  which gets larger as we increase the angle). However, the block remains stationary until we reach a **critical angle**  $\theta_c$ . For angles greater than  $\theta_c$ , the block will accelerate down the plane. What does this say about the force of friction for angles less than  $\theta_c$ ? This means that the force of friction must also increase as the angle increases in order to balance the component of the force in the positive x-direction. If the object is **stationary**, the frictional force adjusts to balance the other horizontal forces acting on the object. This frictional force is usually called the **static** friction.

## Kinetic Friction

We noted above that as we increase the inclination angle, there will be some point where the component of the force in the direction of motion becomes larger than the force of friction. This means that there must be some **maximum value for the** magnitude of static friction between two given objects. Once this maximum value is exceeded, the object begins to move down the plane. You would think that at the critical angle, the force of friction would be balanced by the force acting down the plane. In this case, the object would move down the plane at a constant velocity. However, this is **not** what is usually observed. What is observed is that the object accelerates down the plane. This is because the force of friction actually **decreases** somewhat, once the object begins to move over the surface. We call the force of friction on a moving object its **kinetic friction**. **Relationship between the Kinetic Frictional Force and the Normal Force** The force of friction depends on the contact forces between the object and the surface that it moves upon. We call this force of contact the **normal force**. The **normal force is always perpendicular to the surface** of



motion. While the force of kinetic friction can be quite complicated, it has been found that in most cases, the **magnitude** of the Kinetic Frictional Force is simply proportional to the **magnitude** of the Normal Force. Therefore, we can write

$$F_f = \mu_k N$$

where  $\mu_k$  is the **coefficient of kinetic friction**. The coefficient of kinetic friction is simply a number, (i.e. it is dimensionless), that depends on the nature of the surface and object.

**The Force of Kinetic Friction is always in the opposite direction of the motion.**

### Relationship between the Static Frictional Force and the Normal Force

The force of static friction also depends on the Normal force. Since the static force can change depending on the applied force, it is difficult to write an equation that applies in all cases. However, there is a relationship between the normal force and the **maximum** force of static friction. This relationship is

$$F_{s(\max)} = \mu_s N,$$

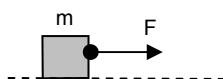
where  $F_{s(\max)}$  means the maximum static frictional force and  $\mu_s$  is the coefficient of static friction.

### The Coefficient of Kinetic and Static Friction

The coefficients of friction are numbers which are determined from experiment. They are **material parameters**, which means that the number we get depends on what the material we consider. We can't say that the coefficient of friction for steel is 0.53 or the coefficient for glass is 0.27. This is because  **$\mu_k$  depends on both the object and the planar surface**. The coefficient of steel sliding on a piece of glass is not the same as for steel on wood. The coefficient of friction also depends on the **surface textures** of the object and plane. The coefficient of static friction also depends on the materials and surface textures of the objects in contact. Since we know that the force of friction decreases somewhat when the object is in motion, we know that

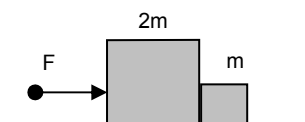
$$\mu_k < \mu_s$$

#### Example(4.2.1):



If an object of mass  $m$  on a frictionless horizontal surface is horizontally pulled by a force of  $F$  as shown in the figure on the left, its acceleration is  $3m/s^2$ . If the surface were frictional and the coefficient of friction were 0.1, what would the acceleration of the object be in  $m/sec^2$ ?

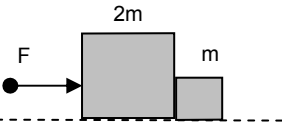
#### Example(4.2.2):





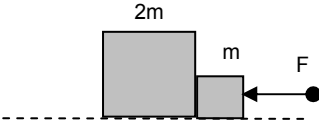
If an objects of masses  $2m$  and  $m$  on a frictionless horizontal surface are horizontally pushed by a force of  $F$  as shown in the figure on the left , what is the magnitude of the interaction force between objects in terms of  $F$  ?

Example(4.2.3):



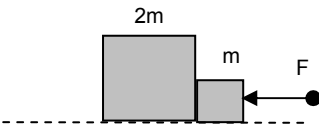
Objects of masses  $2m$  and  $m$  on a frictional horizontal surface are horizontally pushed by a force of  $F$  as shown in the figure on the left . If the coefficient of friction (  $k$  ) between the objects and horizontal surface is  $0.2$  , what is the magnitude of the interaction force between objects in terms of  $F$  ?

Example(4.2.4):



If objects of masses  $2m$  and  $m$  on a frictionless horizontal surface are horizontally pushed by a force of  $F$  as shown in the figure on the left , what is the magnitude of the interaction force between objects in terms of  $F$  ?

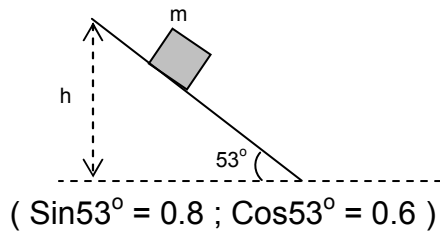
Example(4.2.5):



Objects of masses  $2m$  and  $m$  on a frictional horizontal surface are horizontally pushed by a force of  $F$  as shown in the figure on the left . If the coefficient of friction (  $k$  ) between the objects and horizontal surface is  $0.2$  , what is the magnitude of the interaction force between objects in terms of  $F$  ?



Example(4.2.6):



An object of mass  $m$  on a frictional inclined plane is free to slide down along the inclined surface as shown in the figure on the left . If the coefficient of friction (  $k$  ) between the object and horizontal surface is  $0.1$  ;

a) What is the magnitude of normal force applied by the inclined surface on the object in terms of  $mg$  ?

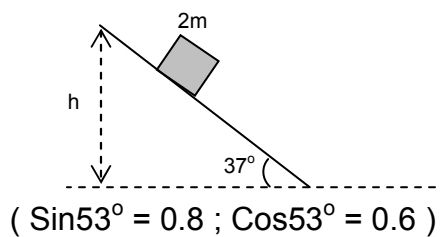
b) What is the magnitude of frictional force acting on the object along the inclined surface in terms of  $mg$  ?

c) What is the magnitude of net force acting on the object along the inclined surface in terms of  $mg$  ?

d) Does the object slide down along the inclined surface ?

e) What is the magnitude of the acceleration of the object along the inclined surface in terms of  $g$  ?

Example(4.2.7):



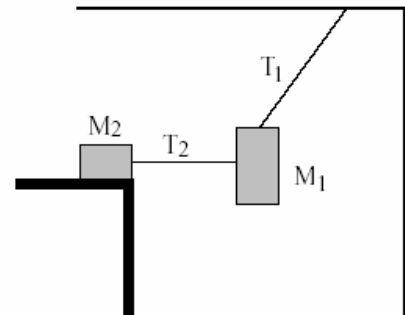
An object of mass  $2m$  on a frictional inclined plane is free to slide down along the inclined surface as shown in the figure on the left . If the coefficient of friction (  $k$  ) between the object and horizontal surface is  $0.1$  ;



- a) What is the magnitude of normal force applied by the inclined surface on the object in terms of  $mg$  ?
- b) What is the magnitude of frictional force acting on the object along the inclined surface in terms of  $mg$  ?
- c) What is the magnitude of net force acting on the object along the inclined surface in terms of  $mg$  ?
- d) Does the object slide down along the inclined surface ?
- e) What is the magnitude of the acceleration of the object along the inclined surface in terms of  $g$  ?

### An Example of Static Friction

A mass  $M_1$  is suspended from 2 cords as in the figure on the right. The cord from the ceiling is at an angle of  $30^\circ$  to the vertical. Mass  $M_2$  has a mass of 20 kg and the static coefficient of friction between  $M_2$  and the surface is 0.5. What is the largest hanging mass that can be supported by this system? ( $\sin 30 = 0.5$ ,  $\cos 30 = 0.8$ )



**Step 1.** The hanging mass can be supported as long as  $M_2$  stays fixed. We start with a free body diagram for  $M_2$ . As long as the tension  $T_2$  in the string is less than  $F_{s(max)}$ ,  $M_2$  will remain fixed.

Therefore,

$$T_{2(max)} = \mu N = \mu M_2 g = (0.5)(20)(10) = 100 \text{ N}$$

**Step 2:** We draw a free body diagram for the hanging mass  $M_1$ . We know that in the x-direction

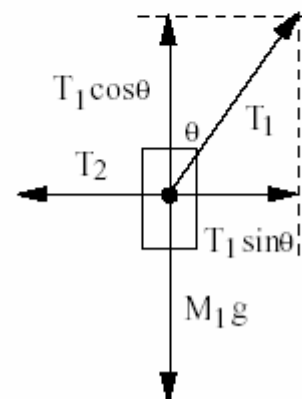
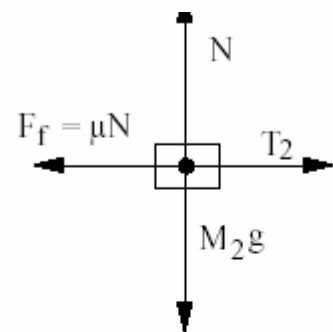
$$T_{2(max)} = T_{1(max)} \sin 30 = T_{1(max)} (0.5)$$

$$\text{Or } T_{1(max)} = 2.0 \times T_{2(max)}$$

For the y-components

$$M_{1(max)} g = T_{1(max)} \cos 30 =$$

Substituting in our values, we can solve for  $M_{1(max)}$





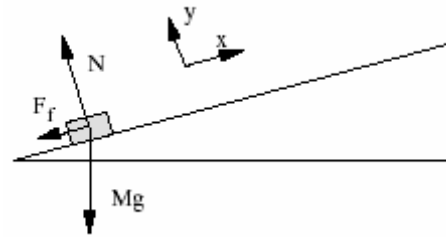
$$M_{1(\max)} = 2.0 (100 \text{ N})(0.8)/(10 \text{ m/s}^2) = 16.0 \text{ kg}.$$

**An Example of Kinetic Friction.**

An object of mass 10 kg at the base of an inclined plane is projected up the plane with an initial velocity of 10 m/s. The angle of inclination is 30° and the coefficient of friction  $\mu_k$  is 0.35. How far does the object travel up the plane?

**Step 1:** Draw a diagram of the problem and find all the forces.

If the object is moving in the positive x direction, the frictional force is in the direction down the plane.



**Step 2:** Break up the gravitational force into its components along the x and y direction. Then write down Newton's Laws for the forces.

$F_y = N - Mg \cos \theta = 0$  since No acceleration in y-direction

so

$$N = Mg \cos \theta$$

In the x-direction

$$F_x = -Mg \sin \theta - \mu N$$

(Both forces acting in the -x direction)

$$F_x = -Mg \sin \theta - \mu Mg \cos \theta = -Mg(\sin \theta + \mu \cos \theta)$$

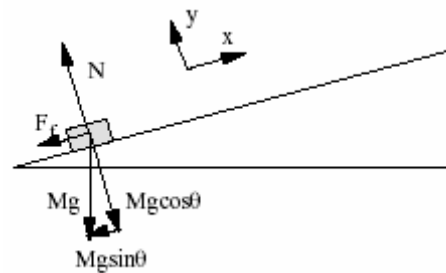
But  $F_x = Ma$ , so

$$Ma = -Mg(\sin \theta + \mu \cos \theta)$$

or

$$a = -g(\sin \theta + \mu \cos \theta) = -10.0(\sin 30 + 0.35 \cos 30) = -10.0(0.80) = -8.03 \text{ m/s}^2$$

(Acceleration is minus because object is slowing down)



Now we know the acceleration. At the top, the velocity will be zero. So we can use  $v = v_0^2 + 2ax$

Let's make the substitutions  $v=0$ ,  $v_0=10 \text{ m/s}$ , and  $a = -5.85 \text{ m/s}^2$



$$0 = 100 - 2(8.03) x$$
$$\text{or, } x = (100 \text{ m}^2/\text{s}^2)/(11.7 \text{ m/s}^2) = \mathbf{6.22 \text{ m}}$$

## **AIR FRICTION**

### **Drag Forces: Another Type of Frictional Force**

The equation for the frictional force  $F = \mu N$  does not work in all cases. This form of the frictional force works well for solid surfaces in contact. It does not describe the frictional or drag forces of objects moving through a fluid. Air is a special fluid, and the drag force caused by the atmosphere is sometimes called air resistance. The drag force for objects moving at relatively high velocities has been determined by experiment to be:

$$D = k A v^2$$

where  $k$  is the drag coefficient and it depends on the density of the medium,  $A$  is the cross-sectional area of the object and  $v$  is the object's velocity. The drag force is an example of a velocity dependent force. If an object is dropped, it begins to accelerate under the gravitational force. Since the initial velocity is small, the drag force doesn't have a large effect on the motion. As the speed of the object increases, so does the drag force. At some point, the drag force will become as large as the gravitational force. At this point, the net force is zero and the object continues its descent at a constant velocity. This velocity where the drag force is equal to the gravitational force is called the **terminal velocity**. From Newton's second law, we can determine the terminal velocity,  $v_t$ , for an object falling in a fluid. Since the acceleration is zero, we know that

$$mg = D$$

or

$$mg = k A v^2$$

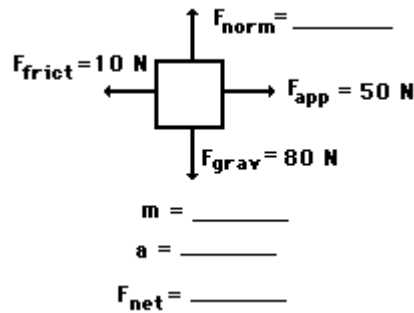
We can solve for  $v$  (Terminal velocity)



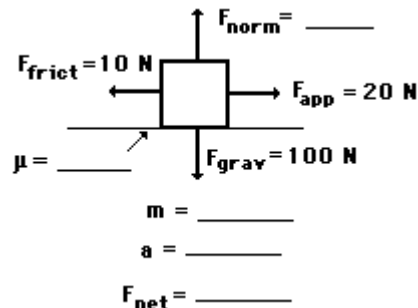
$V = \sqrt{\frac{mg}{kA}}$	Terminal Velocity
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An applied force of 50 N is used to accelerate an object to the right across a frictional surface. The object encounters 10 N of friction. Use the diagram to determine the normal force, the net force, the mass, and the acceleration of the object. (Neglect air resistance.)



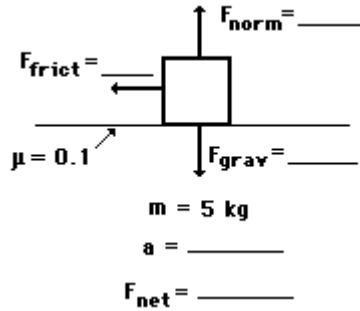
An applied force of 20 N is used to accelerate an object to the right across a frictional surface. The object encounters 10 N of friction. Use the diagram to determine the normal force, the net force, the coefficient of friction ( $\mu$ ) between the object and the surface, the mass, and the acceleration of the object. (Neglect air resistance.)



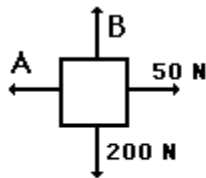
A 5-kg object is sliding to the right and encountering a friction force which slows it down. The coefficient of friction (" $\mu$ ") between the object and the surface is 0.1. Determine the force of gravity, the normal force,



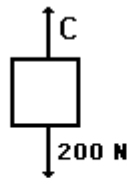
the force of friction, the net force, and the acceleration. (Neglect air resistance.)



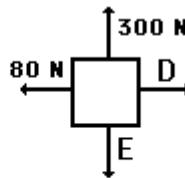
Free-body diagrams for four situations are shown below. The net force is known for each situation. However, the magnitudes of a few of the individual forces are not known. Analyze each situation individually and determine the magnitude of the unknown forces.



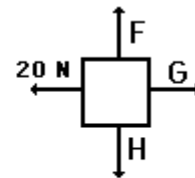
$F_{\text{net}} = 0 \text{ N}$



$F_{\text{net}} = 900 \text{ N, up}$



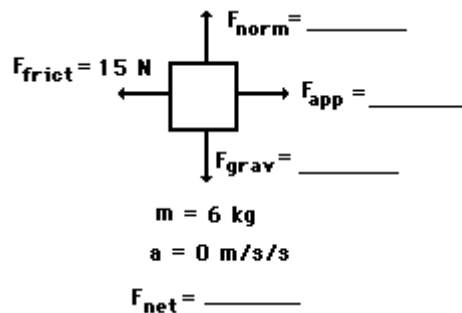
$F_{\text{net}} = 60 \text{ N, left}$



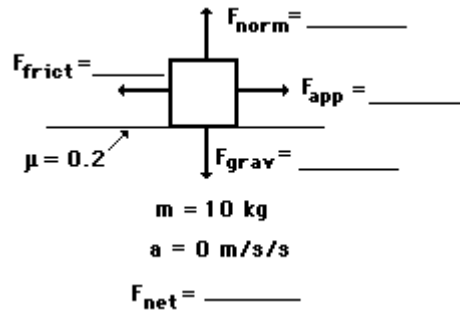
$F_{\text{net}} = 30 \text{ N, right}$



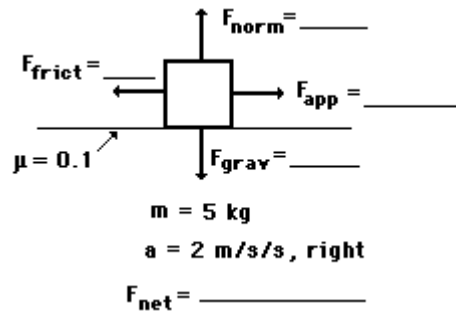
A rightward force is applied to a 6-kg object to move it across a rough surface at constant velocity. The object encounters 15 N of frictional force. Use the diagram to determine the gravitational force, normal force, net force, and applied force. (Neglect air resistance.)



A rightward force is applied to a 10-kg object to move it across a rough surface at constant velocity. The coefficient of friction between the object and the surface is 0.2. Use the diagram to determine the gravitational force, normal force, applied force, frictional force, and net force. (Neglect air resistance.)



A rightward force is applied to a 5-kg object to move it across a rough surface with a rightward acceleration of 2 m/s/s. The coefficient of friction between the object and the surface is 0.1. Use the diagram to determine the gravitational force, normal force, applied force, frictional force, and net force. (Neglect air resistance.)



A rightward force of 25 N is applied to a 4-kg object to move it across a rough surface with a rightward acceleration of 2.5 m/s/s. Use the diagram to determine the gravitational force, normal force, frictional force, net force, and the coefficient of friction between the object and the surface. (Neglect air resistance.)

