



CHAPTER 4

MOTION IN A STRAIGHT LINE



AIM OF THE CHAPTER:

To learn about;

1. Motion in a straight line
2. Difference between velocity and speed
3. Define acceleration
4. Analyze motion graphs



KEYWORDS

Displacement Position
Average Speed
Average Velocity
Average acceleration
Instantaneous acceleration

Every object in the universe is in state of motion. The objects may be in different states of motions. They may translate, rotate or vibrate. The motion of an object may be one dimensional along a straight line, two dimensional in a plane or three dimensional in space. The subject of motion seems to be complicated, but you will see that all the different motions can be described by using a few simple equations. The branch of mechanics which deals with the description of motion is called kinematics.

In this chapter, we will define various concepts related to motion such as, position, displacement, average velocity, instantaneous velocity, speed, and acceleration. Then, we will explain one dimensional motion with constant velocity and constant acceleration. One dimensional motion is quite common in nature. For example, the motion of objects falling near the surface of the earth is a one dimensional motion with constant acceleration. Similarly, the projectile motion occurring in a plane can be treated as the combination of two one dimensional motions along horizontal and vertical directions.

POSITION AND DISPLACEMENT

Position

The term position refers to the location of an object. The position of an object can be specified by stating its distance and direction relative to a reference point. For example, point A is located 2m east of point O. This statement indicates that, the position of point A relative to point O has magnitude of 2 m and direction of eastward.

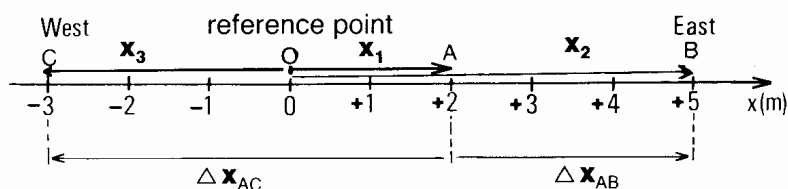


Figure 1

The position of any point on this coordinate axis may be represented by a number x , which indicates how far the point is from the origin. The sign of the number indicates the direction of the position vector. If the sign is positive, the point is located on the right of the origin and its position vector is directed along the positive x -axis. If the sign is negative, the point is located on the left of the



origin and its position vector is directed along the negative x-axis. For example, the positions of the points A, B and C, denoted by vectors \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_3 in Fig 1, are represented by the numbers +2, +5 and -3 (in m), respectively. Here, the minus sign indicates that point C is located on the negative x-axis, 3 meters left of point O. Note that, the numbers representing the positions of these points would be different if we would select another point as the origin (reference point).

DISPLACEMENT

Suppose that an object at position x_1 , at time t_1 is found at another position x_2 at a later time t_2 . We say that the object changed its position during the time interval from t_1 , to t_2 . **The change in the position of the object ($x_2 - x_1$), is called the displacement of the object.** We use the symbol Δ (delta) to indicate the change in a quantity. Thus, the change in position, called displacement, is

$\Delta X = (x_2 - x_1)$	Displacement
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AVERAGE SPEED

The speed of a moving object is the distance the object moves per unit of time. Speed is **a scalar quantity**; it indicates how fast the object has moved but does not tell anything about the direction of motion. If the total distance travelled by an object is divided by the elapsed time we obtain the average speed of the object.

Average speed (V) = $\frac{\text{total distance travelled}}{\text{elapsed time}}$

The speed of an object is constant if it moves equal distances in each successive unit of time. In this case, the speed at any time is equal to the average speed. The SI unit of speed is meter per second (m/s).



Refer to Fig 1

a) An object at point A moved to point B, find its displacement.

b) Another object at point A moved to point C, find its displacement.



A car travels a distance of 432 km in 8-h . What is the average speed of the car in

- (a) km/h and
- (b) m/s?



AVERAGE VELOCITY AND INSTANTANEOUS VELOCITY

AVERAGE VELOCITY:

Suppose that an object is displaced from position x_1 , to position x_2 during the time interval

$(t_2 - t_1)$. **The average velocity is defined as the displacement per unit time,**

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow \bar{v} = \frac{\Delta x}{\Delta t} \quad \text{Average Velocity}$$



Refer to the Fig 3 below.

An object starts from point O and moves to point A in 12 s. It then travels back to point B in 8 s. Find the average

- a) during the first 12 s, (b) during the last 8 (c) for the whole trip. (d) What is the average speed of the object for the whole trip?

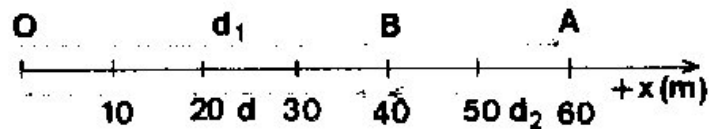


Figure 2



A runner makes one lap around a 450 m track in a time of 50 sec. What were the runner's (a) average speed, and (b) average velocity?



A student starts from his home and walks 300 m east and 400 m north to get to his school. The total trip takes 10 minutes. Find his (a) average speed and (b) average velocity.



ONE DIMENSIONAL MOTION WITH CONSTANT VELOCITY

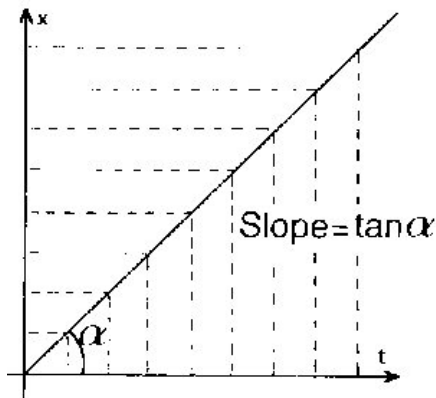
If the displacement of an object is equal in each successive unit of time, then the object moves with constant velocity along a straight line. For this uniform motion, the displacement of the object for a given time interval is proportional to the time interval.

If we assume that the object starts from the origin ($x_1 = 0$) at $t_1 = 0$ and let $X_2 = x$ and $t_2 = t$, we obtain,

$$X_2 - X_1 = V (t_2 - t_1),$$
$$\Delta X = V \Delta t$$

$$X = V.t \quad \text{Constant velocity}$$

The position (x) versus time (t) graph of an object moving with constant velocity is a straight line passing through the origin. The slope of this line gives the constant velocity.



Position versus time graph. The graph for constant is a straight line. Equal distances are covered in equal time intervals.

Figure 4

The velocity versus time graph of an object moving with constant velocity is a straight line parallel to the time axis. The area under this graph, between t_1 and t_2 is equal to the displacement in this time interval.

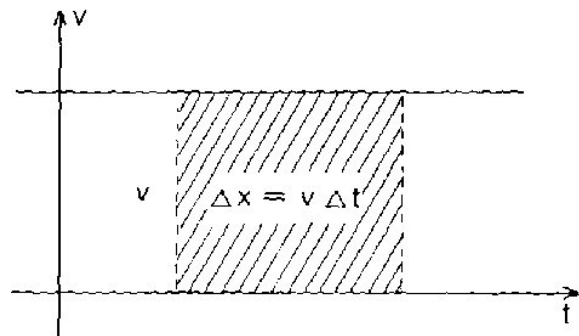


Figure 5



An object moving with constant velocity covers a 40 m distance in 8 s.

- a) Find the distance traveled in 15 s.
- b) Find the time for the object to travel 65 m distance.



Consider a plane flying at a constant velocity of 60 m/s.

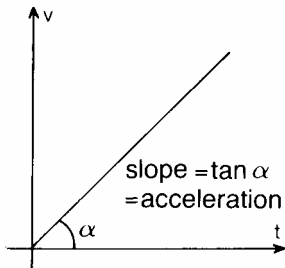
- a) Tabulate the distance it travels in one, two, three, four and five seconds.
- b) Plot its position vs. time graph. Is the graph straight line.
- c) Find the slope of the line. What does the slope represents.
- d) Plot velocity vs. time graph.
- e) Calculate the area under the $V - t$ graph between $t_1=0$ and $t_2=5$ sec.



AVERAGE ACCELERATION

When the velocity of an object changes with time the object is said to be accelerating. **The change in velocity per unit time is called acceleration.** Suppose that the velocity of an object is \mathbf{v}_1 , at t_1 , and \mathbf{v}_2 at a later time t_2 . The average acceleration during the time interval $(t_2 - t_1)$ is defined as,

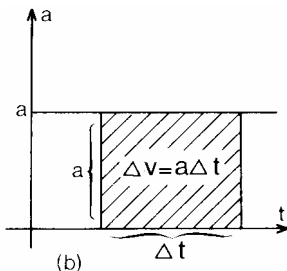
$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$



The average acceleration is a vector quantity along the direction of $\Delta \mathbf{v}$. Its magnitude is equal to $\Delta v / \Delta t$ and has a unit of meter per second squared (m/s^2).

Figure 6a

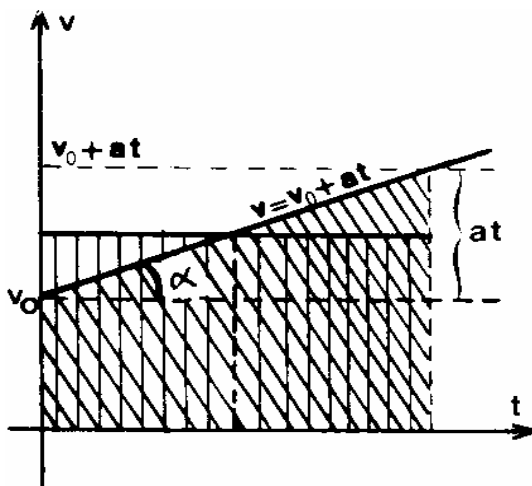
If the change in velocity is the same for each successive unit of time the object moves with constant acceleration. For this uniform change in velocity (constant acceleration) the velocity versus time graph is a straight line (Fig 6a). The slope of this line is constant and equal to the acceleration.



The graph of this constant acceleration versus time is shown in Fig 6b. We can state that the area under this graph for the time interval 0 to t is equal to the change in velocity.

Figure 6b

ONE DIMENSIONAL MOTION WITH CONSTANT ACCELERATION



In the previous section we have stated that if the velocity of an object changes uniformly its acceleration will be constant. We know that when acceleration is constant the velocity versus time graph is a straight line. The slope of this line is equal to the acceleration (Fig 8). When acceleration is constant the velocity changes each second by the amount of acceleration. Therefore, the change in velocity in time t will be (at) . Here, (a) denotes the acceleration. If we add the change in velocity to the initial velocity (V_0) we obtain the final velocity of the



object after time t.

$$V = V_0 + at$$

Since the change in velocity is uniform, the average velocity in a time interval can be obtained from the mean value of the initial and final velocities.

$$\bar{V} = \frac{V_0 + V}{2}$$

This equation can be used to find the displacement of the object in time t. Suppose that the object starts from the origin ($x_0 = 0$) at $t = 0$. We can find its displacement or position at a later time as ,

$$\begin{aligned} X &= X_0 + \bar{v} t \\ &= 0 + \bar{v} t \\ X &= \frac{V_0 + V}{2} t \end{aligned}$$

From Fig 8, you can see that the rectangular areas under the average velocity versus time or the trapezoid area under velocity versus time graphs are equal to the displacements expressed by the above equations. Therefore, **the area under v-versus t-graph for a given time interval is equal to the displacement of the object for that interval.**

$$\begin{aligned} X &= \frac{V_0 + V}{2} t \\ &= \frac{V_0 + (V_0 + at)}{2} t \\ X &= V_0 t + \frac{1}{2} at^2 \end{aligned}$$

This equation is plotted in Fig 9. As you can see from the figure, **the position versus time (x versus t) graph for constant acceleration is a parabola.**

If we eliminate t between these equations.

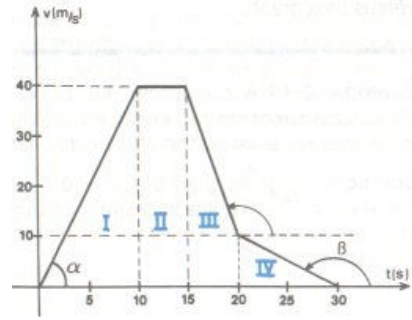
$$\begin{aligned} t &= \frac{V - V_0}{a} \\ X &= \frac{V_0 + V}{2} t \\ &= \left(\frac{V_0 + V}{2} \right) \left(\frac{V - V_0}{a} \right) \\ &= \frac{V^2 - V_0^2}{2a} \end{aligned}$$

$$V^2 = V_0^2 + 2ax$$



A car accelerates along a straight road from rest to 54 km/h in 6 s. What is the average acceleration of the car?

b) Plot acceleration vs. time graph.



At $t_i = 2$ s an object is traveling at 4 m/s. At $t_f = 5$ s its velocity is -2 m/s. Find the average acceleration for the given time interval.

A block starts from rest and slides down on an inclined plane. It is 4 m away from the starting point at the end of the 4 sec. Assume that the acceleration is uniform, find:

- its acceleration
- its velocity at the end of the 4 sec

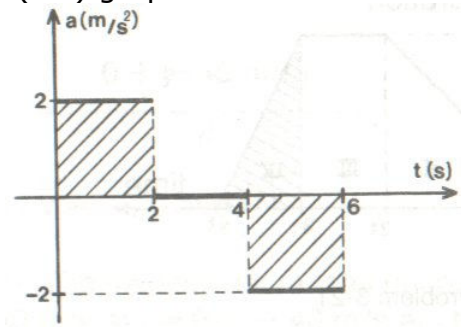


The velocity vs. time graph of a moving object is given.

- Find the acceleration of the object during the first 10 sec and also last 10 sec.
- What is the average acceleration during the interval between the tenth and the fifteenth seconds?



(a-t) graph of an object is given. Assume that $X_0=0$ and $V_0=0$ when $t=0$. Plot (V-t) graph for $t=0$ and $t=6$.



An object starts from rest with a constant acceleration of $8 m/s^2$ with a constant acceleration. Find:

- the speed at the end of the 5 sec.
- The average speed for the 5 sec
- Distance traveled in 10 sec



A car starts from rest and reaches a velocity of $20 m/s$ in 10 sec. It then continues with this velocity for two minutes and approaches an intersection, driver applies the brakes for 6 sec and stops at the intersection.

- Plot (V-t) graph
- Plot (a-t) graph
- Find the displacement for the last 6 sec.
- What is the total distance traveled?



A truck increases its speed from $4 m/s$ to $16 m/s$ in 20 sec. Determine:

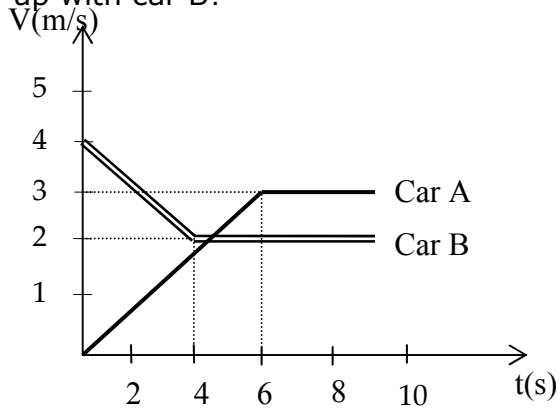
- the average speed
- the acceleration
- Distance traveled in 8 sec.



A smart bus moving at a speed of $20 m/s$ begins to slow down at a constant rate of $3 m/s^2$. Find how far it goes before stopping.



(V-t) graph for the cars A and B is given. When the car A will catch up with car B?



A car moving at 30 m/s slows down to a speed of 10 m/s in a time of 5.0 sec. Determine:
a) the acceleration of the car
b) the distance traveled within the third second.



A skier starts from rest and slides 9 m down a slope in 3 sec. In what time after starting will the skier have a speed of 24 m/s. Assume that the acceleration is constant.



Velocity vs. time graph of an object is given.

- a) Plot (a-t) graph
- b) Plot (x-t) graph

