CHAPTER 2
FORCE, TORQUE, EQULIBRIUM

AIM OF THE CHAPTER:

To learn about;

1. Force
   - concurent-coplanar forces
2. Torque
3. Equilibrium
   - Free-body diagram

KEYWORDS

Force	lever arm
Concurent forces	Moment arm
Paralel forces	Equilibrium
Resultant force	Torque
Pivot point	
Fulcrum	
Free-body diagram	

FORCE

We experience **force** as any kind of a push or a pull on an object. When you push a car, you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We say that an object falls because of the *force of gravity.* Force do not always give rise to notion. For example, you may push very hard desk and it may not move.

A force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition that we have discussed before.

We can represent any force on a diagram by an arrow. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force. The SI unit for force is called Newton (N).

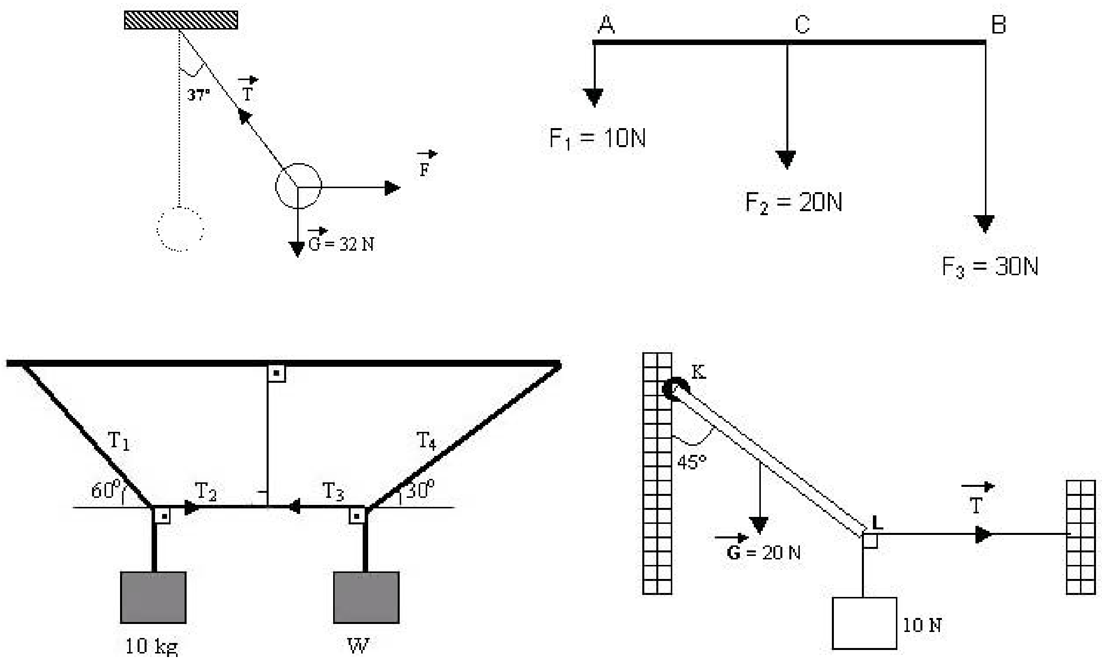
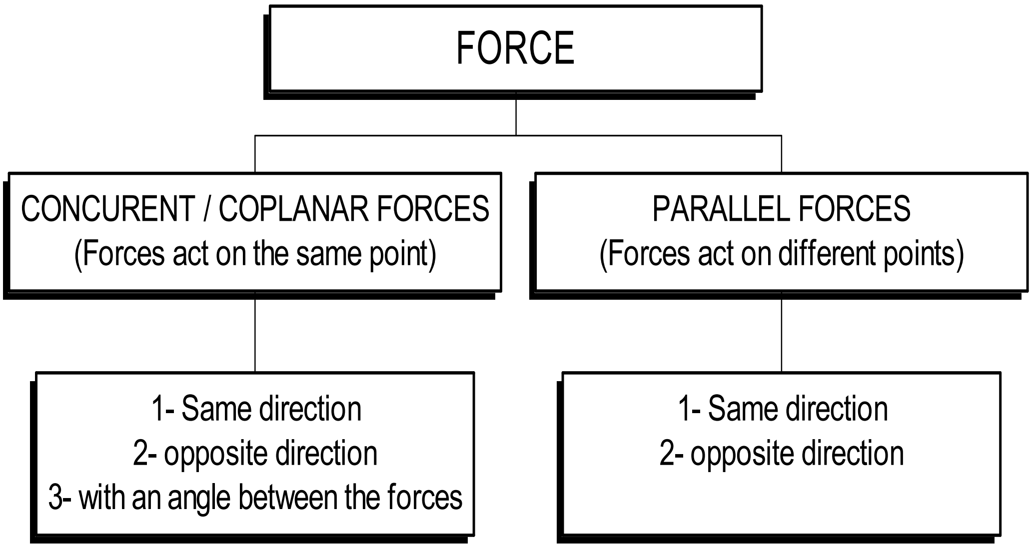


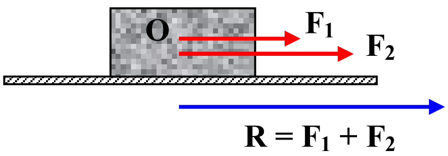
Fig 1.1. Force representations in different physical systems.

We can divide forces into following catagories:



Now let us try to investigate the properties of these cases.

1- CONCURENT FORCES – SAME DIRECTION:

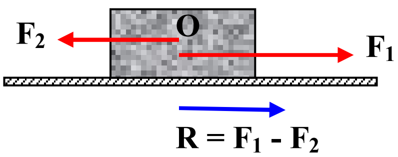


Forces acting on point O : $\mathbf{F_1}$ and $\mathbf{F_2}$
Magnitude of the resultant force:

$$R = F_1 + F_2$$

Direction of the resultant force: Same with the forces
Starting point of the resultant force: Same with the forces – point O.

2- CONCURENT FORCES – OPPOSITE DIRECTION:

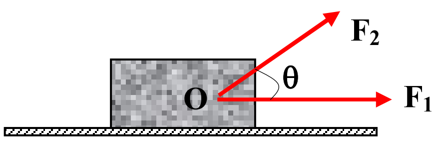


Forces acting on point O : $\mathbf{F_1}$ and $\mathbf{F_2}$
Magnitude of the resultant force:

$$R = F_1 - F_2$$

Direction of the resultant force: Same with the bigger force
Starting point of the resultant force: Same with the forces – point O.

3- CONCURENT FORCES – WITH AN ANGLE BETWEEN THEM:



If we have two forces F_1 and F_2 with an angle of θ between them, methods that we have disscussed for vectors are still appliable.

a) Parallelogram method: The forces are drawn from the same beginning point to form the adjacent sides of a parallelogram as shown in **fig 1-2a**. The parallelogram is then completed by drawing parallel lines to the two forces $\mathbf{F_1}$ and $\mathbf{F_2}$. The diagonal drawn from the beginning of the force to the opposite corner of the parallelogram is the force **R** representing the sum of $\mathbf{F_1}$ and $\mathbf{F_2}$ **fig 1-2b**.

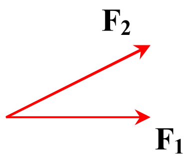


fig 1-2a

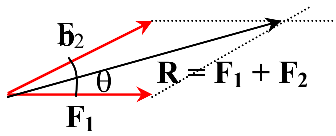


fig 1-2b

$$c^2 = a^2 + b^2 + 2\,a\,b\,\mathrm{Cos}\theta$$

b) Head-to-tail: One of the forces, **a** and **b,** is moved parallel to itself and they are drawn head-to-tail as shown in **Fig 1-3a-b.** Neither the direction nor the length of the force is changed during this drawing. A third force **R** is drawn from the tail of first force to the head of the second force. Force **R** represents the sum of the resultant of forces $\mathbf{F_1}$ and $\mathbf{F_2}$.

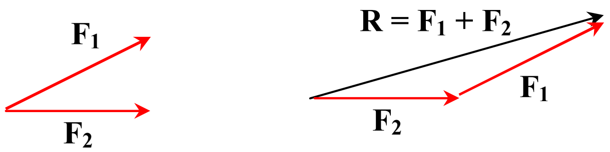


Fig.1-3a

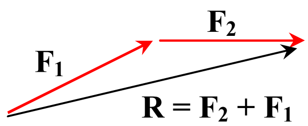
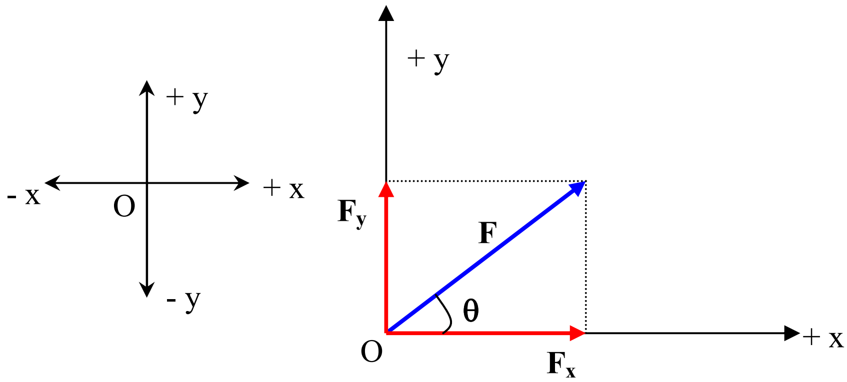


Fig.1-3b

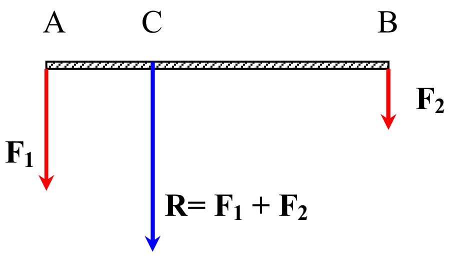
c) Resolving into components: It is always possible to resolve a force into two components along any given pair of perpendicular directions. In order to resolve a force into its own components we need a suitable cordinate system. We generally define the horizontal axis as x-axis and vertical axis as y-axis. Any given force **F** can be drawn with the proper angle θ such that its tail coincides with the origin of the axes as shown. The components along the x- and y-axes of this force is usually called $\mathbf{F_x}$ and $\mathbf{F_y}$ respectively**.** To find these components draw pependicular lines from the head of r to x and y axes. The magnitude of these components are then find using the suitable trigonometric equations.



$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$
$$F = \sqrt{F_x^2 + F_y^2}$$
$$\tan\theta = \frac{F_y}{F_x}$$

1- PARALLEL FORCES – SAME DIRECTION

Assume we have two force namely F_1 and F_2 acting on the different points A and B respectively along the same direction. Magnitude, direction, and the location of the resultant force can be find by the following procedure.



RESULTANT FORCE:
Magnitude: Addition of the forces

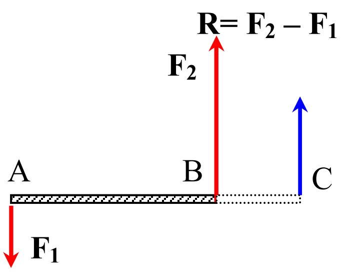
$$R = F_1 + F_2$$

Direction: Same with the forces.
Location: **Always** between the two force and **always** close to the bigger force. Location of the resultant force can be find from:

$$F_1|AC| = F_2|BC|$$

2- PARALLEL FORCES – OPPOSITE DIRECTION

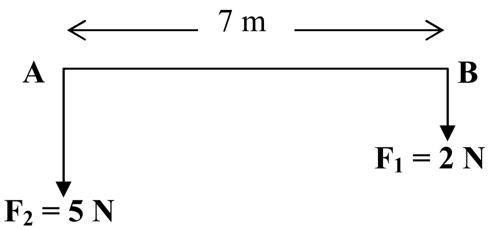
Assume we have two force namely F_1 and F_2 acting on the different points A and B respectively along the opposite direction. Magnitude, direction, and the location of the resultant force can be find by the following procedure.



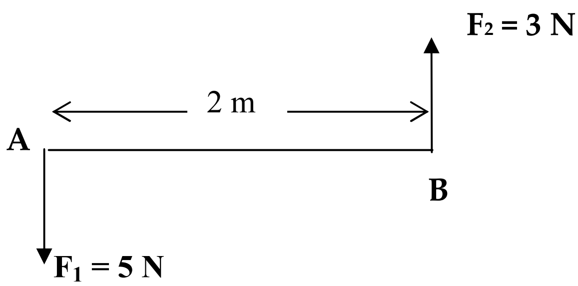
RESULTANT FORCE:
Magnitude: $R = F_2 - F_1$
Direction: Same with the bigger force.
Location: **Always** outside of the starting points of the forces and **always** closer to the bigger force. Location of the resultant force can be find from:

$$F_1|AC| = F_2|BC|$$

Find the position and the magnitude of the resultant force.



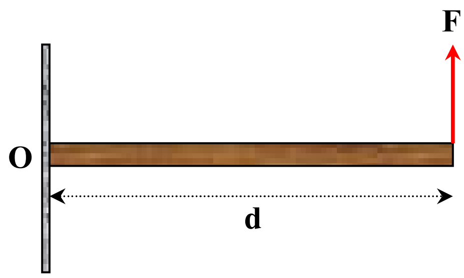
Find the position and the magnitude of the resultant force.



TORQUE

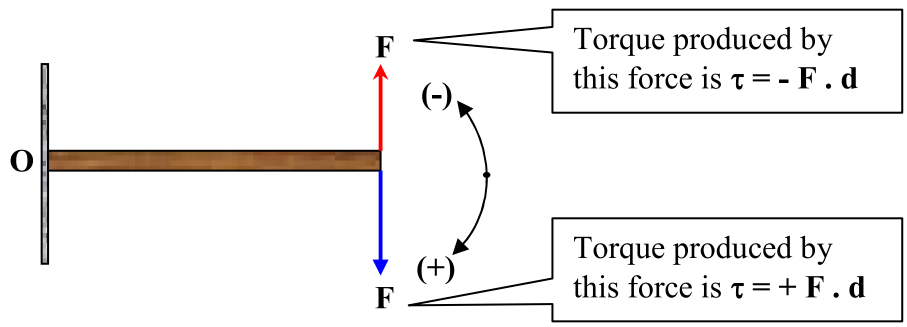
To make an object start rotating about an axis clearly requires a force. But the direction of this force, and where it is applied, are also important. Take, for example, an ordinary situation such as the door given in the figure. If you apply a force $\mathbf{F_1}$ perpendicular to the door as shown, you will find out the greater the magnitude, $\mathbf{F_1}$, the more quickly the door opens. (We ignore the friction) But now if you apply the same magnitude of force at a point closer to the hinge, say $\mathbf{F_2}$, you will find that the door will not open so quickly. Indeed, it is found that turning effect produced on the door is not only proportional to the magnitude of the force, but it is also directly proportional to the perpendicular distance from *the axis of rotation to the line along which force acts.* This distance d is called the **lever arm** or **moment arm** of the force and point O is called **pivot** or **fulcrum**.

Torque is a vector quantity and we are going to represent torque by a Greek letter τ (Tau)



τ = Force . distance
$\tau = F \cdot d$
τ = Torque = N.m
F = Force = Newtons
d = Distance = meters

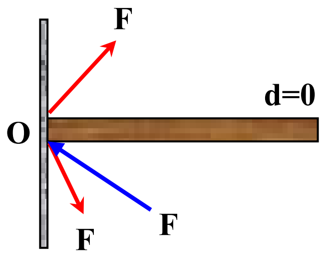
Direction of torque is specified as *clockwise* or *counterclockwise.* Usually *clockwise* direction is accepted as positive (+) and *counterclockwise* as negative (-).



SPECIAL CASES:

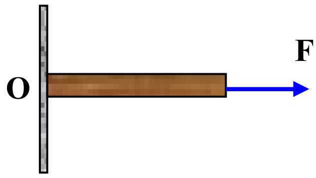
1- If force is zero then torque will be zero. F = 0 then τ **= 0**

2- If the distance is zero then torqu will be zero. d=0 then τ **= 0**

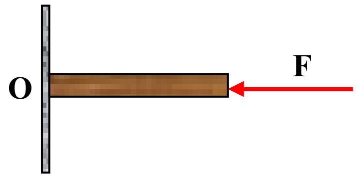


$$\tau = 0$$

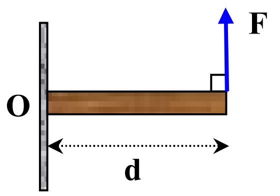
3- If the force acting on the beam is passing through thr application point, O, in this case torque produced by the force will be zero.



$$\tau = 0$$



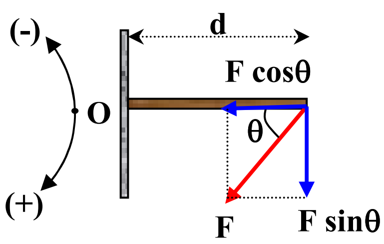
4- If the force is acting perpendiculary, torque produced will be:



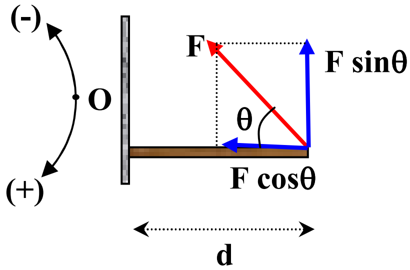
$$\tau = F.\ d$$

5- If the force is acting on the beam with an angle of θ, resolve the force into components and perpendicular force will be responsible for the torque produced.

In the below picture force **F** is divided into two components. Since the cos component is passing through the application point, torque produced by this force will ve zero. Net torque acting on the system will be:

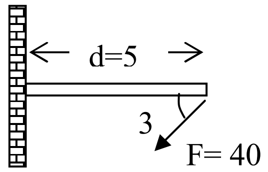


$$\tau = +\ F.\sin\theta$$

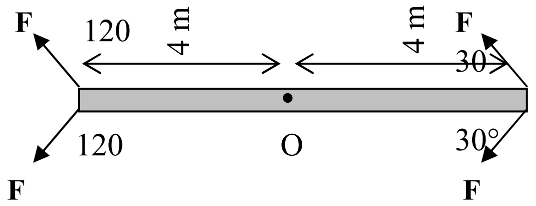


$$\tau = -\ F.\sin\theta$$

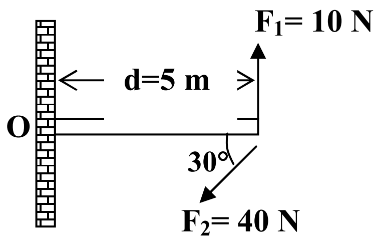
Find the torque produced by the force. (Sin 30 = ½)



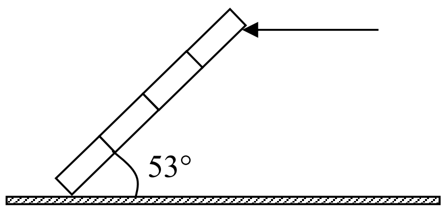
In the above given figure, in which direction beam will rotate?



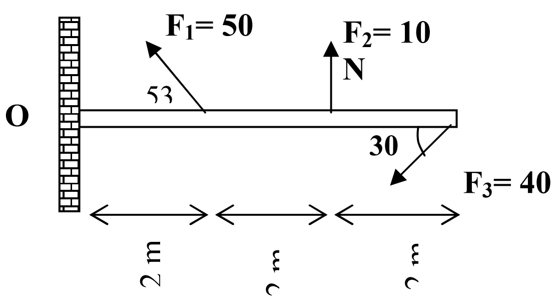
Calculate the total torque and direction of the motion around point O.(Sin 30 = ½)



I f the below given system is in equilibrium and the weightof the beam is 80 N, calculate the force F. (Sin 30 = ½ , Cos 37=0.8, Sin 37 = 0.8)



Calculate the total torque and direction of the motion around point O.
(Sin 30 = ½ , Cos 37=0.8, Sin 37 = 0.8)



RIGID OBJECTS IN EQUILIBRIUM

If a rigid body is in equilibrium, its motion does not change. By "motion" we mean both linear and rotational motion. An object whose motion is not changing has no acceleration of any kind. Therefore, the net force ΣF applied to the object must be zero, since $\Sigma F = ma$ and $a = 0$. For two-dimensional motion, the condition $\Sigma F=0$ means that the x and y components of the net force are separately zero: $\Sigma F_x=0$ and $\Sigma F_y = 0$. When calculating the net force, we include only *external forces,* that is, those forces applied to the object by external agents. We ignore the forces that one internal part of an object exerts on another into part. Internal forces can be ignored because they occur in action-reaction which have no effect as far as the motion of the entire object is concerned. Each action-reaction pair consists of oppositely directed forces of equal magnitude, the effect of one force cancels the effect of the other.

A net external torque causes rotational motion to change. But there is no change in the motion of a rigid body in equilibrium, so there can be no net torque under equilibrium conditions. The sum of the positive torques must be balances the sum of the negative torques. Using the symbol $\Sigma\tau$ to represent the net external torque (the sum of all positive and negative torques), we write this conditions

$$\Sigma\tau = 0$$

EQUILIBRIUM OF A RIGID BODY

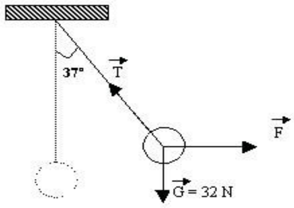
A rigid body is in equilibrium if it has zero translational acceleration and angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero:

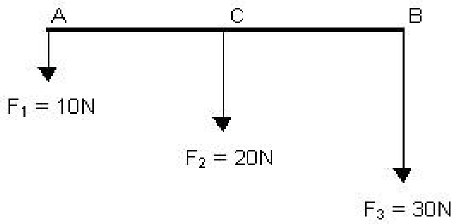
$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$
$$\Sigma\tau = 0$$

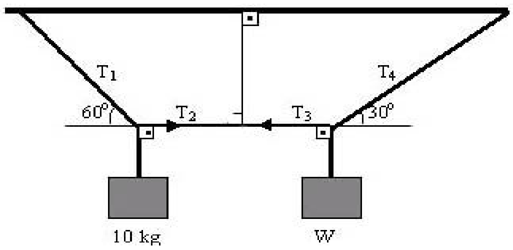
http://www.mta.ca/faculty/science/physics/ntnujava/forceDiagram/forceDiagram.html

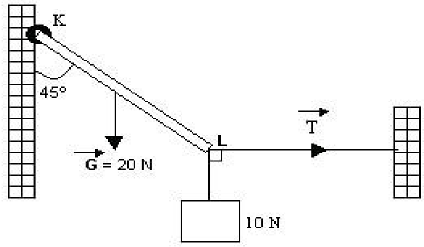
Applying the conditions of equilibrium to a rigid body

1- Select the object to which conditions for equilibrium are to be applied.
2- Draw a free-body (free-body diagram = is a diagram that represents the object and all forces acting on the object) diagram that shows all the external forces acting on the object, each force with a proper direction
3- Choose a convenient set of x, y axes and resolve all forces into components that lie along these axes.
4- Apply the conditions that specify the balance of forces at equilibrium: $\Sigma F_x = 0$ and $\Sigma F_y = 0$
5- Select a convenient axis of rotation. Identify the point where each extarnal force acts on the object, and calculate the torque produced by each force about axis of rotation. Set the sum of the torques about this axis equal to zero. $\Sigma\tau = 0$
6- Solve the equations in steps 4 and 5 for the desired unknown quantities.





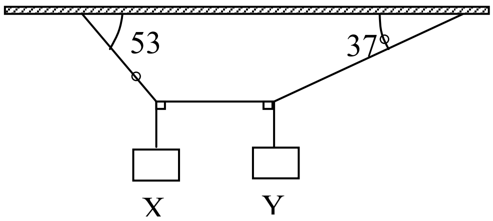


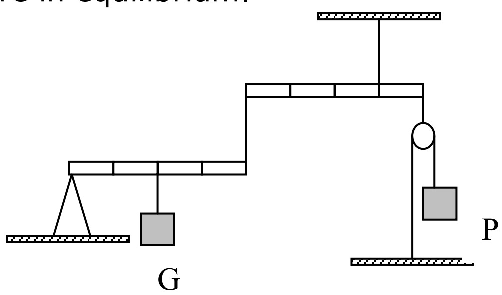


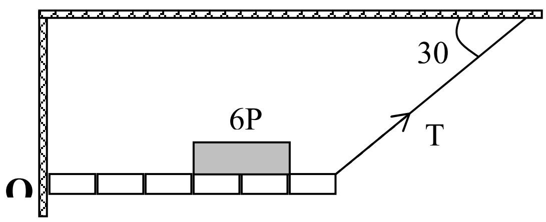
Free-body diagrams and suitable coordinate systems.

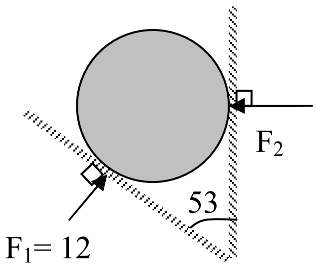
For the above given system describe the forces acting on the system and choose a suitable coordinate system for each case.

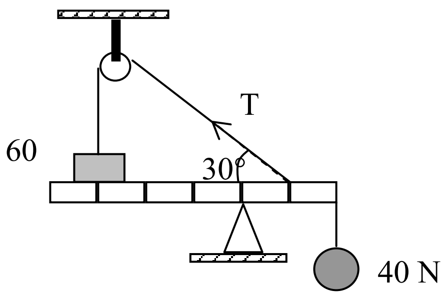
Draw the free-body diagram for the following physical systems. Assume that all the systems are in equilibrium.



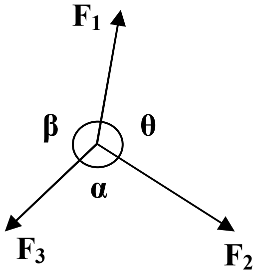






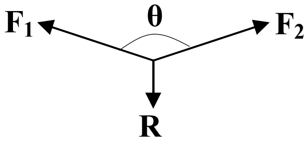


Another very useful equation for three forces F_1, F_2, and F_3 acting on the same point with different angles is called sine rule.



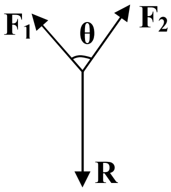
1- $R = F_1 + F_2 + F_3$, $|R| = 0$
2- $F_1 + F_2 = -F_3$
3- $F_1 + F_3 = -F_2$
4- $F_2 + F_3 = -F_1$
5- $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\theta}$

If the angle between two force increases, resultant force of these two will decrease.



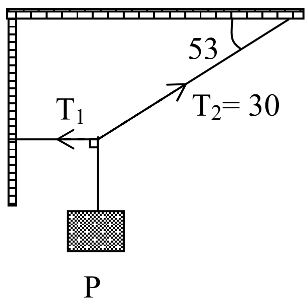
$\theta\uparrow$ then $R\downarrow$

If the angle between two force decrease, resultant force of these two will increase.



$\theta\downarrow$ then $R\uparrow$

If the below given system is in equilibrium then find the value of P.
(Sin 30 = ½ , Cos 37=0.8, Sin 37 = 0.8, Sin 53=0.6)



If the tension on the horizontal string is 80 N then find the weight W.
(Sin 30 = ½ , Cos 37=0.8, Sin 37 = 0.8, Sin 53=0.6)