Chapter 2: Describing Motion:

Kinematics in One Dimension

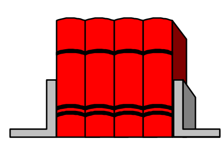
Outline

Major Concepts

By the end of the chapter, students should understand each of the following and be able to demonstrate their understanding in problem applications as well as in conceptual situations.

- Reference frames
- Position, distance, and displacement
- Speed and velocity
  - Average
  - Instantaneous
  - Constant
- Acceleration
  - Average
  - Instantaneous
  - Constant
- Equations of motion with constant acceleration
- Free fall
- Graphs of position versus time, velocity versus time, and acceleration versus time

Book References:



Physics: Principles with Applications, Giancoli, Sixth Edition Chapter 2 pages 19-44.

AIM OF THE CHAPTER:

To learn about;

1. Motion in a straight line
2. Difference between velocity and speed
3. Define acceleration
4. Analyze motion graphs

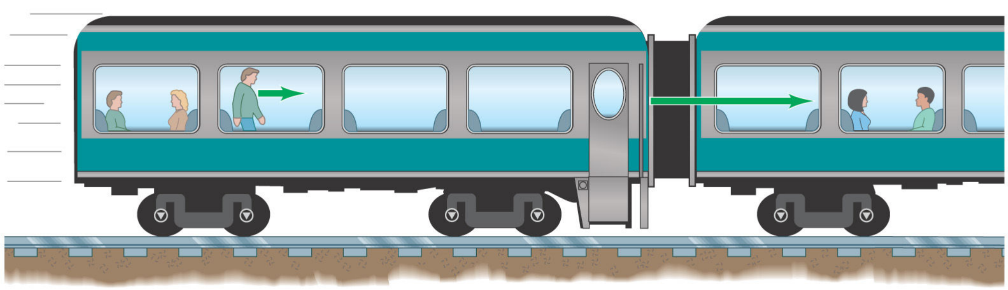
KEYWORDS

Displacement
Position
Average Speed
Average Velocity
Average acceleration
Instantaneous acceleration

Week 3: Lesson 1

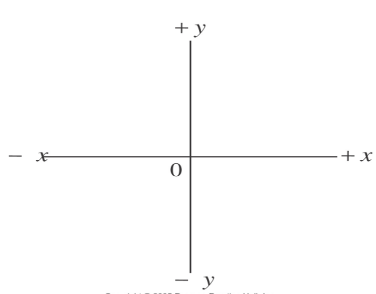
2.1 Reference Frames and Displacement

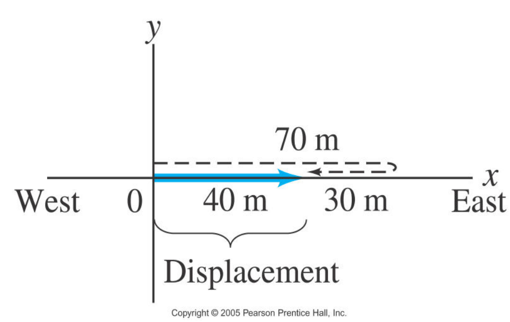
Any measurement of position, distance, or speed must be made with respect to a reference frame.
For example, if you are sitting on a train and someone walks down the aisle, their speed with respect to the train is a few miles per hour, at most. Their speed with respect to the ground is much higher.



A **scalar** quantity has only magnitude, e.g., a car travelling at 90 km/h.
A **vector** quantity has magnitude and direction, e.g., a car travelling at 90 km/h north.

In Physics we often draw a set of **coordinate axes** as shown in the figure below.
We make a distinction between distance and displacement.
Distance traveled (dashed line) is the actual length travelled, so it is a **scalar** quantity.





Displacement (Dark Arrow) is how far the object is from its starting point, regardless of how it got there. It is defined as ***the change in position of the object.***

Displacement is the straight line distance between two points, plus direction, so it is a quantity that has both magnitude and direction. Such a quantities are called **vectors**, and are represented by arrows in diagrams. For example the the arrow in the above figure represents the displacement whose **magnitude** is 40 m and whose **direction** is is to the right (east)

The displacement is written: $\boldsymbol{\Delta x = x_2 - x_1}$

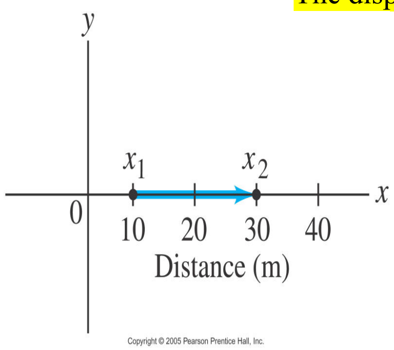


Figure (a)

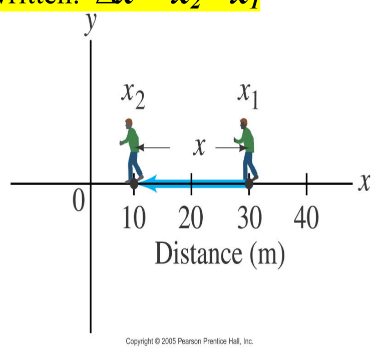


Figure (b)

In **figure (a)** the arrow represents the displacement $\Delta x = x_2 - x_1 = 30.0$ m - 10.0 m = 20.0 m **Displacement is positive.**

In **figure (b)** the displacement $\Delta x = x_2 - x_1 = 10.0$ m - 30.0 m = - 20.0 m, the displacement vector points to the left. **Displacement is negative.**

For one-dimensional motion along the *x*-axis, a vector pointing to the right has a **positive** sign, whereas a vector pointing to the left has a **negative** sign.

2.2 Average Velocity

Speed refers to how far an object travels in a given time interval, regardless of direction. It has magnitude but no direction. Therefore speed is a scalar quantity.
Average speed of an object is defined as ***the total distance traveled along its path divided by the time it takes to travel this distance:***

$$\text{average speed} = \frac{\text{Distance traveled}}{\text{time elapsed}}$$

$$S = \frac{X_{total}}{\Delta t}$$

Velocity is used to signify both the ***magnitude*** (numerical value) of how fast an object is moving and also the ***direction*** in which it is moving. Therefore velocity is a vector quantity.

Average velocity is defined in terms of displacement:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{Final position} - \text{initial position}}{\text{time elapsed}}$$

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

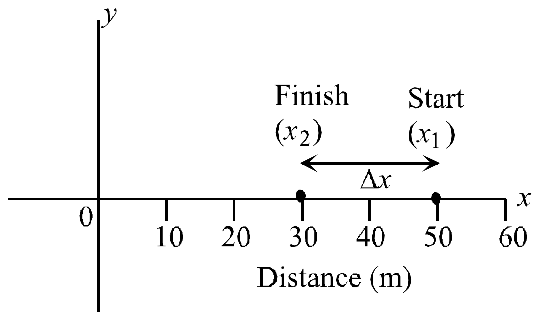
Average speed and average velocity have the same magnitude when the motion **is all in one direction.**

For the usual case of the + x axis to the right, note that if x_2 is less than x_1, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the + x axis and negative for an object moving to the left.

The direction of average velocity is always the same as the direction of the displacement.

Example 2-1

The position of a runner as a function of time is plotted as moving along the x-axis of a coordinate system. During a 3.00 s time interval, the runner's position changes from x_1 = 50.0 m to x_2 = 30.5 m, as shown in the figure below. What was the runner's average velocity?



Solution:

The displacement is $\Delta x = x_2 - x_1 = 30.5\,\text{m} - 50.0\,\text{m} = -19.5\,\text{m}$.

The elapsed time, or time interval, is Δt = 3.00 s. The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\,\text{m}}{3.00\,\text{s}} = -6.50\,\text{m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the x axis. Thus we can say that the runner's average velocity is 6.50 m/s to the left.

Example 2-2

How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

Solution:

$$\Delta x = \bar{v}\Delta t = (18\,\text{km/h})(2.5\,\text{h}) = 45\,\text{km}.$$

2.3 Instantaneous Velocity

The instantaneous velocity is the average velocity, in the limit as the time interval becomes infinitesimally short.

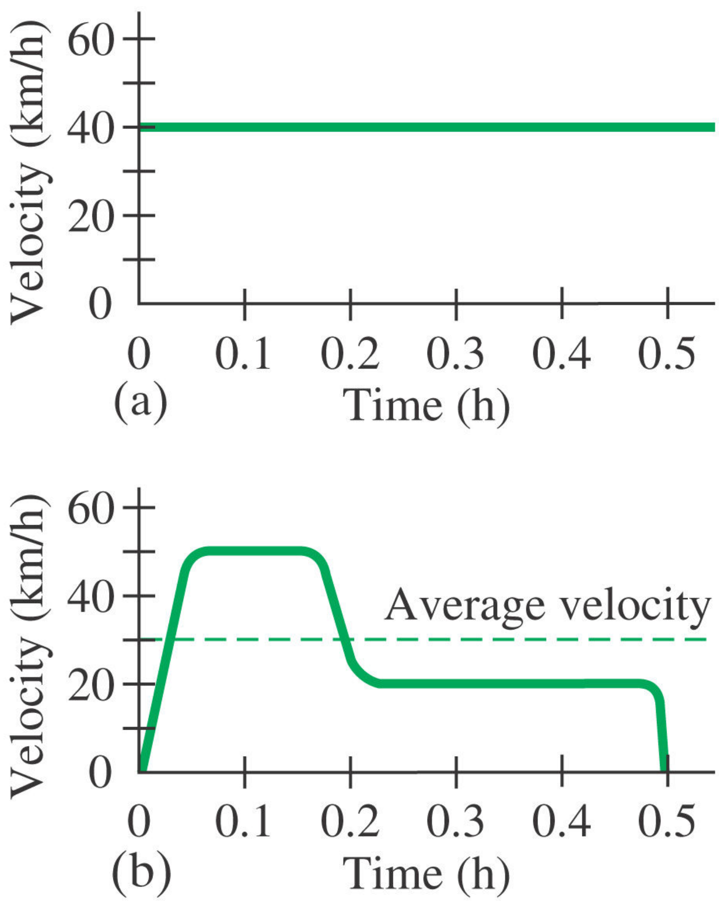
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

For instantaneous velocity we use the symbol v.

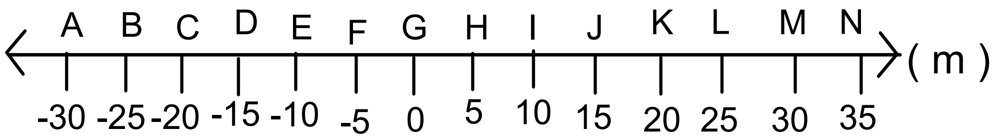
Note that the *instantaneous speed* always equals the magnitude of the *instantaneous velocity*. Why? Because the distance and the magnitude of the displacement become the same when they become infinitesimally small.

If an object moves at a uniform (constant) velocity during a particular time interval, then the its *instantaneous velocity* at any instant is the same as its average velocity. But in many situations this is not the case.

These graphs below show the velocity of a car as a function of time: (a) constant velocity and (b) varying velocity.



Example:



A car travels from the point **B** to the point **F** in 10 seconds .

a) What is its initial position ?

b) What is its final position ?

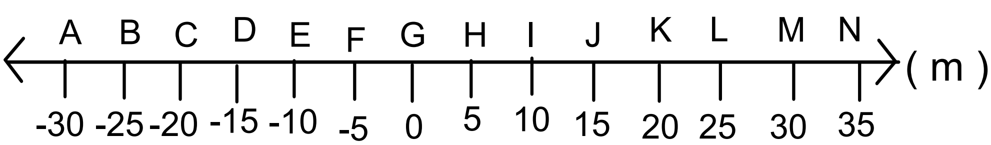
c) What is its displacement ?

d) What is the distance traveled by it ?

e) What is the average velocity of the object ?

f) What is the average speed of the object ?

Example:



A car travels from the point **L** to the point **G** in 5 seconds .

a) What is its initial position ?

b) What is its final position ?

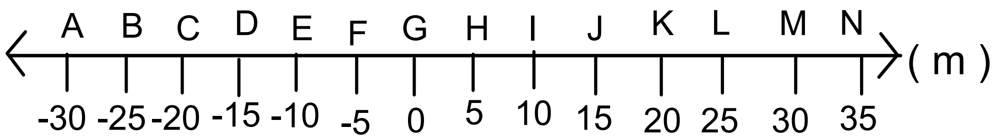
c) What is its displacement ?

d) What is the distance traveled by it ?

e) What is the average velocity of the object ?

f) What is the average speed of the object ?

Example:



A car travels from the point **B** to the point **M** in 6 seconds, then, from the point **M** to the point **E** in 4 seconds.

a) What is its initial position ?

b) What is its final position ?

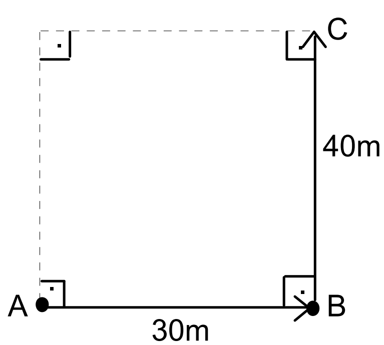
c) What is its total displacement ?

d) What is the total distance traveled by it ?

e) What is the average velocity of the object ?

f) What is the average speed of the object ?

Example:



A car travels 30m toward east in 15 seconds. Then, it travels 40m toward north in 10 seconds.

a) What is the total distance traveled by it ?

b) What is its total displacement ?

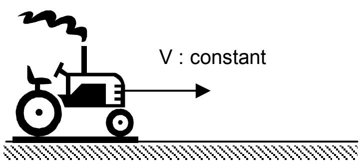
c) What is its average velocity

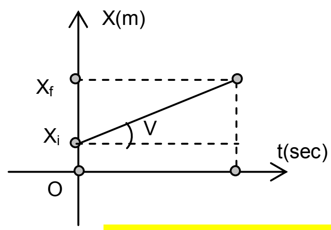
Motion with Constant Velocity

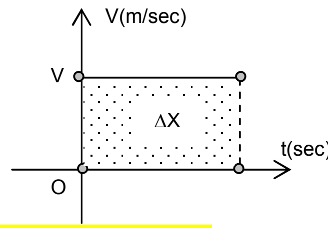
$$V = \frac{\Delta X}{\Delta t}$$

If the displacement of an object is equal in each successive unit of time, then the object moves with constant velocity along a straight line. For this uniform motion, the displacement of the object for a given time interval is proportional to the time interval.

- If the object is moving along the + **x** direction:

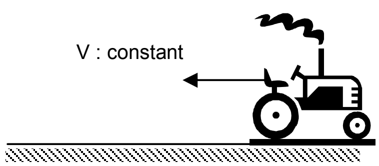


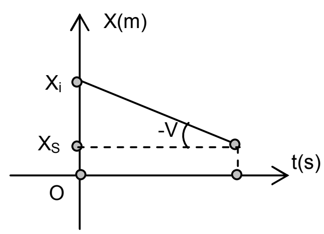


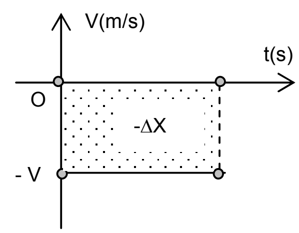


- the **slope** of **(x-t)** graph is **Velocity**.
- the **area** under the **(v-t)** graph is **displacement** along the positive x axis.

- If the object is moving along the **- x** direction:

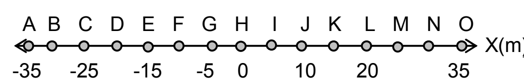






- the **slope** of **(x-t)** graph is **velocity. (negative)**
- the **area** under the **(v-t)** graph is displacement along the negative axis.

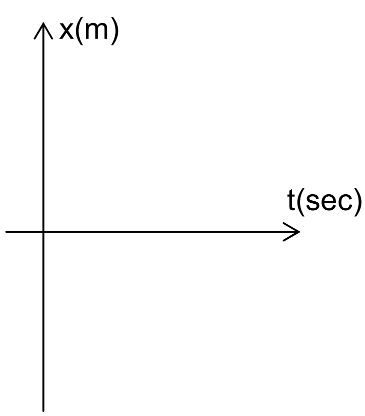
Example:

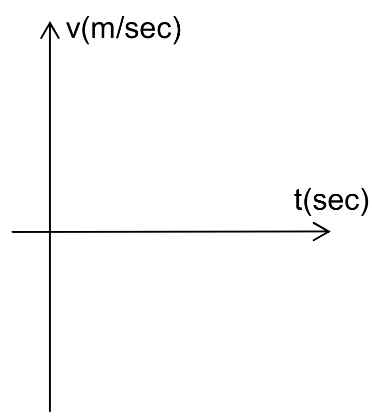


If an object moves with a constant velocity from the point **B** to the point **K** in 15 sec ;

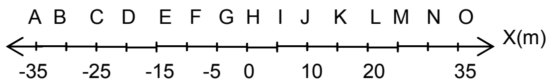
a) Draw its **(x-t)** graph .

b) Draw its **(v-t)** graph .





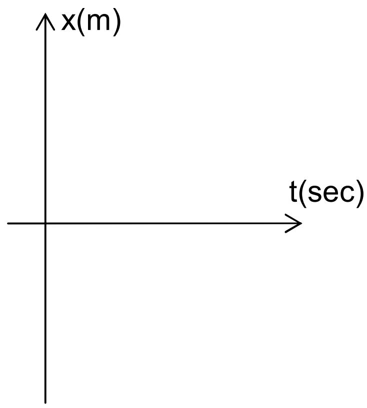
Example:

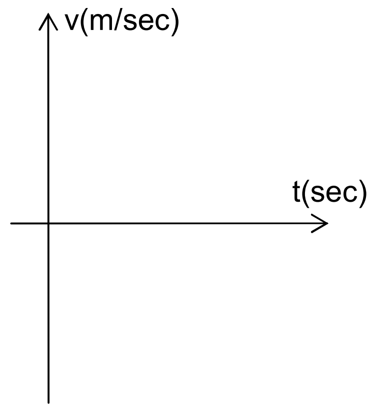


An object moves with a constant velocity from the point **C** to the point **N** in 5 seconds .Then , it moves with a constant velocity from the point **N** to the point **E** in 15 seconds ;

a) Draw its x-t graph.

b) Draw its v-t graph





2.4 Acceleration

An object whose velocity is changing is said to be accelerating. **Average acceleration** is defined as the change in velocity divided by the time taken to make this change:

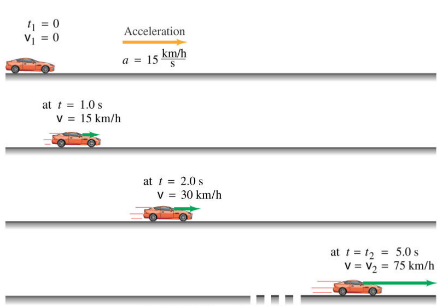
$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time for change to occur}}$$

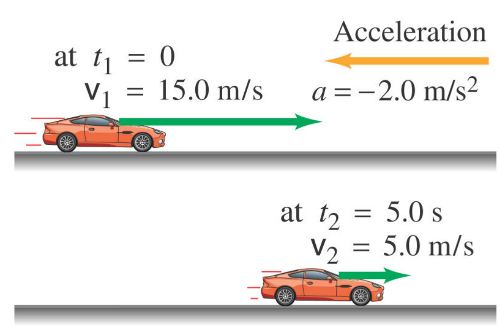
Average acceleration : $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

Acceleration is a vector, although in one-dimensional motion we only need the sign. The units for acceleration are; m/s^2

The instantaneous acceleration, a, can be defined in analogy to instantaneous velocity, for any specific instant:

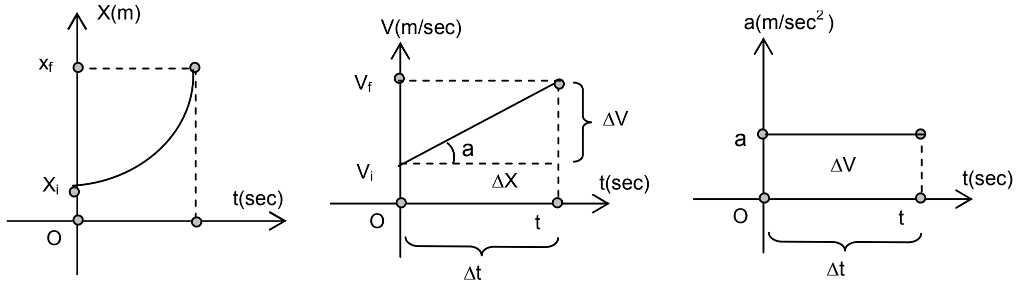
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$





The figure above shows positive acceleration and negative acceleration.:

(x-t), (V-t) , and (a-t) graphs of the motion is shown below.



The **area** under the **(v-t)** graph is **displacement** of the object.
The **slope** of the **(v-t)** graph is the **acceleration** of the object.
The **area** under the **(a-t)** graph is the **change in velocity** (ΔV)

http://www.walter-fendt.de/ph14e/acceleration.htm

Example 2-3

A car accelerates along a straight road from rest to 75 km/h in 5.0 s. What is the magnitude of its average acceleration?

Solution:

$$\text{Average acceleration} = \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{75\,\text{km/h} - 0\,\text{km/h}}{5.0\,\text{s}} = 15\frac{\text{km/h}}{\text{s}}.$$

Conceptual Example 2-4

(a) If the velocity of an object is zero, does it mean that the acceleration is zero? **(b)** If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

Solution:

A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (a) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but he acceleration is not zero since the velocity of the car changes. (b) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0$ $v \neq 0$.

Example 2-5

An automobile is moving to the right along a straight highway, which we choose to be the positive x-axis. Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is v_1 = 15.0 m/s, and it takes 5.0 s to slow down to v_2 = 5.0 m/s, what was the car's average acceleration?

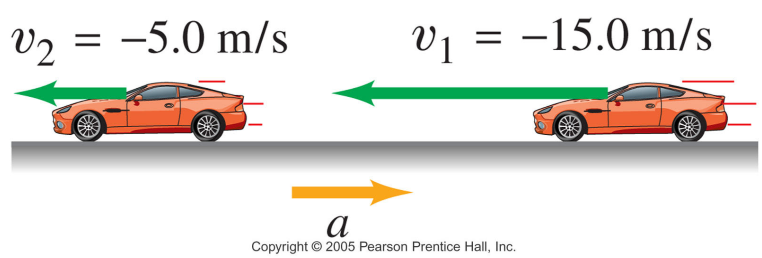
Solution:

$$\bar{a} = \frac{5.0\,\text{m/s} - 15.0\,\text{m/s}}{5.0\,\text{s}} = -2.0\,\text{m/s}^2$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left even though the velocity is always pointing to the right (see the above figure).

Deceleration:

When an object is slowing down, we sometimes say it is **decelerating.** Deceleration means the magnitude of the velocityis decreasing; it does not necessarily mean a is negative.
There is a difference between negative acceleration and deceleration:
Negative acceleration is acceleration in the negative direction as defined by the coordinate system.
Deceleration occurs when the acceleration is opposite in direction to the velocity.



The car of Example 2–5, now moving to the left and decelerating. The acceleration is positive.

Homework 1:

Answer the following questions

1. What must be your car's average speed in order to travel 235 km in 3.25 h?

2. A bird can fly 25 km/h. How long does it take to fly 15 km?

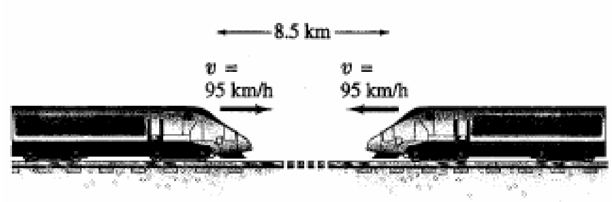
3. If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?

4. Convert 35 mi/h to (a) km/h, (b) m/s, and (c) ft/s.

5. A person jogs eight complete laps around a quarter mile track in a total time of 12.5 min. Calculate (a) the average speed and (b) the average velocity, in m/s.

6. A horse canters away from its trainer in a straight line, moving 116 m away in 14.0 s. It then turns abruptly and gallops halfway back in 4.8 s. Calculate **(a)** its average speed and **(b)** its average velocity for the entire trip, using "away from the trainer" as the positive direction.

7. Two locomotives approach each other on parallel tracks. Each has a speed of 95 km/h with respect to the ground. If they are initially 8.5 km apart, how long will it be before they reach each other? (See Figure shown below).



8. A car traveling 88 km/h is 110 m behind a truck traveling 75 km/h. How long will it take the car to reach the truck?

9. The position of a racing car, which starts from rest at $t = 0$ and moves in a straight line, is given as a function of time in the following Table. Estimate **(a)** its velocity and **(b)** its acceleration as a function of time. Display each in a Table and on a graph.

t (s)	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50
x (m)	0	0.11	0.46	1.06	1.94	4.62	8.55	13.79
t (s)	3.00	3.50	4.00	4.50	5.00	5.50	6.00	
x (m)	20.36	28.31	37.65	48.37	60.30	73.26	87.16	

2.5 Motion at Constant Acceleration

We now examine the situation when the magnitude of the acceleration is constant and the motion is in a straight line. In this case, the instantaneous and average accelerations are equal. We now use our definitons of velocity and acceleration to derive a set of extremily useful equations that relate *x, v, a* and *t* when *a* is constant.

Let us take the initial time to be zero: $t_1 = t_0 = 0$, the initial position and the initial velocity of an object will be represented by x_0 and v_0. At time t the position and velocity will be called *x* and *v*.

The average velocity of an object during a time interval t is: $\bar{v} = \dfrac{x - x_0}{t - t_0} = \dfrac{x - x_0}{t}$

The acceleration, assumed constant, is: $a = \dfrac{v - v_0}{t}$

In addition, as the velocity is increasing at a constant rate, we know that $\bar{v} = \dfrac{v_0 + v}{2}$

Combining these last three equations, we find:

$$x = V_0 t + \frac{1}{2} a t^2$$

We can also combine these equations so as to eliminate t:

$$v^2 = v_0^2 + 2ax$$

We now have all the equations we need to solve constant-acceleration problems.

$v = v_0 + at$	Independent of x
$v^2 = v_0^2 + 2ax$	Independent of t
$x = v_0 t + \frac{1}{2} a t^2$	Independent of v
$x = \frac{v_0 + v}{2} t$	Independent of a

Example 2-6

You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at 2.00 m/s^2. (a) If the runway is 150 m long, can this airplane reach the required speed for take off? (b) If not, what minimum length must the runway have?

Solution:

(a)

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$= 0 + 2(2.0\,\text{m/s}^2)(150\,\text{m}) = 600\,\text{m}^2/\text{s}^2$$
$$v = \sqrt{600\,\text{m}^2/\text{s}^2} = 24.5\,\text{m/s}.$$

This runway length is *not* sufficient.

(b) **Now we want to find the minimum length of runway, $x - x_0$, given v = 27.8 m/s and a = 2.00 m/s^2.**

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\,\text{m/s})^2 - 0}{2(2.0\,\text{m/s}^2)} = 193\,\text{m}.$$

A 200 m runway is more appropriate for this plane.

2-6 Solving Problems

1. Read the whole problem and make sure you understand it. Then read it again.

2. Decide on the objects under study and what the time interval is.

3. Draw a diagram and choose coordinate axes.

4. Write down the known (given) quantities, and then the unknown ones that you need to find.

5. What physics applies here? Plan an approach to a solution.

6. Which equations relate the known and unknown quantities? Are they valid in this situation? Solve algebraically for the unknown quantities, and check that your result is sensible (correct dimensions).

7. Calculate the solution and round it to the appropriate number of significant figures.

8. Look at the result – is it reasonable? Does it agree with a rough estimate?

9. Check the units again.

Example 2-7

How long does it take a car to cross a 30.0 m wide intersection after the light turns green, if the car accelerates from rest at a constant 2.00 m/s^2 ?

Solution:

$$x = \frac{1}{2}at^2,\ t^2 = \frac{2x}{a}$$

$$\text{so } t = \sqrt{\frac{2x}{a}}$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\text{m})}{2.00\,\text{m/s}^2}} = 5.48\,\text{s}$$

Example 2-8

Suppose you want to design an air bag system that can protect the driver in a head-on collision at a speed of 100 km/h (60 mph). Estimate how fast the air bag must inflate to effectively protect the driver. How does the use of a seat belt help the driver?

Solution:

100 km/h = 100 × 10^3 km/3600 s = 28 m/s

$$a = -\frac{v_0^2}{2x} = -\frac{(28\,\text{m/s})^2}{2.0\,\text{m}} = -390\,\text{m/s}^2$$

This enormous acceleration takes place in a time given by

$$t = \frac{v - v_0}{a} = \frac{0 - 28\,\text{m/s}}{-390\,\text{m/s}^2} = 0.07\text{s}$$

To be effective, the air bas would need to inflate faster than this.

What does the air bag do?

It spreads the force over a large area of the chest (to avoid puncture of the chest by the steering wheel). The seat belt keeps the person in a stable position against the expanding air bag.

Example 2-9

Estimate the minimum stopping distance for a car, which is important for traffic safety and traffic design. The problem is best dealt with in two parts, two separate time intervals.

(1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time" during which the speed is constant, so $a = 0$.

(2) The second time interval is the actual braking period when the vehicle slows down ($a \neq 0$) and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about 5.0 m/s^2 to 8.0 m/s^2. Calculate the total stopping distance for an initial velocity of 50.0 km/h (14 m/s) and assume the acceleration of the car is –6.0 m/s^2 (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing). Reaction time for normal drivers varies from 0.30 s to about 1.0 s; take it to be 0.50 s.

Solution:

Part (1): $x_0 = 0$

$x = v_0 t + 0 = (14\ \text{m/s})(0.50\ \text{s}) = 7.0\ \text{m}$

Thus the car travels 7.0 m during the driver's reaction time, until the moment the brakes are applied. We will use this result as input to part (2)

Part (2): **Now consider the second time interval, until the moment the brakes are applied and the car is brought to rest.**

We have initial position $x_0 = 7.0$ m.

$$v^2 - v_0^2 = 2a(x - x_0)$$

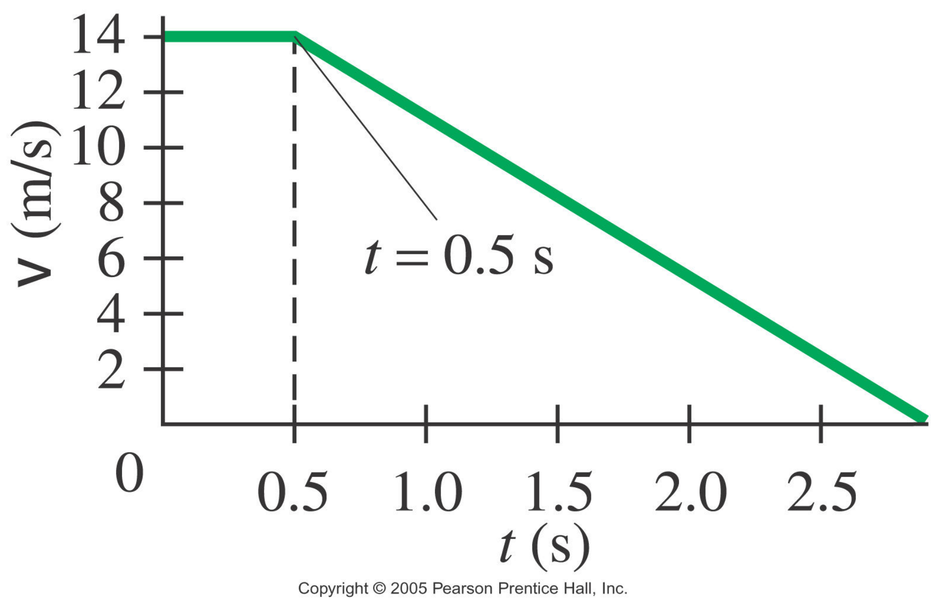
$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 7.0\text{m} + \frac{0 - (14\,\text{m/s})^2}{2(-6.0\,\text{m/s}^2)}$$

$$= 7.0\,\text{m} + \frac{-196\,\text{m}^2/\text{s}^2}{-12\,\text{m/s}^2}$$

$$= 7.0\,\text{m} + 16\,\text{m} = 23\,\text{m}$$

The car travelled 7.0 m while the driver was reacting and another 16 m during the braking period before coming to a stop. The total distance travelled was then 23 m.

Example 2–9. Graph of v vs. t.



Homework 2:

1. A car accelerates from 13 m/s to 25 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

2. A car slows down from 23 m/s to rest in a distance of 85 m. What was its acceleration assumed constant?

3. A light plane must reach a speed of 33 m/s for takeoff. How long a runway is needed if the (constant) acceleration is 3.0 m/s^2?

4. A world-class sprinter can burst out of the blocks to essentially top speed (of about 11.5 m/s) in the first 15.0 m of the race. What is the average acceleration of this sprinter, and how long does it take her to reach that speed.

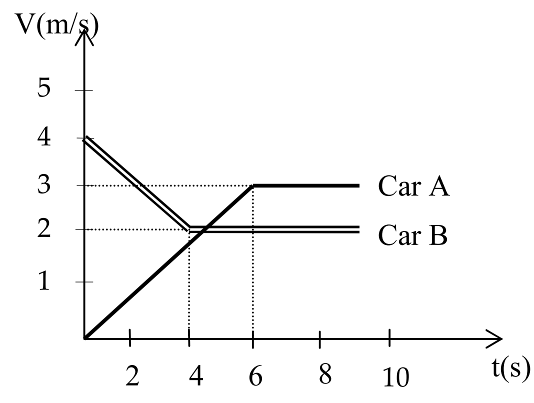
5. A car slows down uniformly from a speed of 21.0 m/s to rest in 6.00 s. How far did it travel in that time?

6. In coming to a stop, a car leaves skid marks 92 m long on the highway. Assuming a deceleration of 7.00 m/s^2, estimate the speed of the car Just before braking.

7. A car traveling 85 km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the average acceleration of the driver during the collision?.

8. Determine the stopping distances for a car with an initial speed of 95 km/h and human reaction time of 1.0 s, for an acceleration (a) $a = -4.0$ m/s^2; (b) $a = -8.0$ m/s^2.

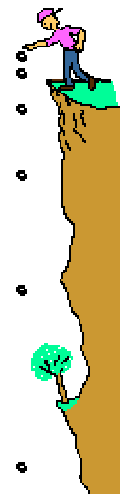
9. (V-t) graph for the cars A and B is given. When the car A will catch up with car B?



10. A car moving at 30 m/s slows down to a speed of 10 m/s in a time of 5.0 sec. Determine:
a) the acceleration of the car
b) the distance traveled within the third second.

2-7 Falling Objects

http://www.walter-fendt.de/ph14e/projectile.htm



A free-falling object is an object, which is falling under the sole influence of gravity. That is to say that any object which is moving and being acted upon only be the force of gravity is said to be "in a state of **free fall**." This definition of free fall leads to two important characteristics about a free-falling object:

- Free-falling objects do not encounter air resistance.
- All free-falling objects (on Earth) accelerate downwards at a rate of approximately 10 m/s^2 (to be exact, 9.8 m/s^2)

The dot diagram at the right depicts the acceleration of a free-falling object. The position of the object at regular time intervals - say, every 0.1 second - is shown. The fact that the distance which the object travels every interval of time is increasing is a sure sign that the ball is speeding up as it falls downward.

The acceleration due to gravity g at the Earth's surface is approximately 9.80 m/s^2. In British units g is about 32 ft/s^2.

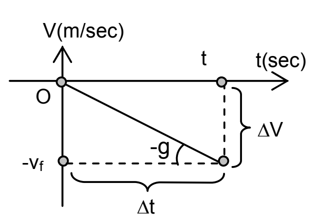
$$V^2 = V_o^2 + 2ax$$
$$V = V_0 + at$$
$$x = V_0 + \frac{1}{2}at^2$$
$$X = \frac{V_0 + V}{2}$$

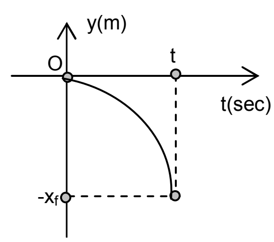
- As we said before for the free fall objects only under the influence of the gravitational acceleration (g =10 m/s^2) so lets replace the acceleration "a" in these formulas by "g"
- Since the object is released from rest so its initial velocity is zero $V_0 = 0$
- Since object is falling down along the y plane so lets replace "x" by "y"

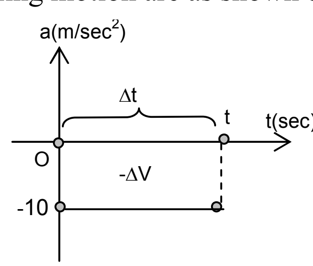
So the new formulas for the free fall only becomes as:

$$V = g\,t$$
$$y = \frac{1}{2} g\,t^2$$
$$y = \frac{V_0 + V}{2}$$

The **(V-t), (y-t)** and **(a-t)** graphs of the object making free falling motion are as shown below







Example 2-10

Suppose that a ball is dropped ($v_0 = 0$) from a tower 70.0 m high. How far will the ball have fallen after a time t_1= 1.00 s, t_2 = 2.00 s, and t_3 = 3.00 s ?

Solution:

$$t = t_1 = 1.00\,\text{s}$$

$$y_1 = v_0 t_1 + \frac{1}{2}at_1^2 = 0 + \frac{1}{2}at_1^2 = \frac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 4.90\,\text{m}$$

The ball has fallen a distance of 4.90 m during the time interval t = 0 to t1 = 1.00 s. Similarly, after 2.00 s (= t2), the ball's position is

$$y_2 = \frac{1}{2}at_2^2 = \frac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 19.6\,\text{m}$$

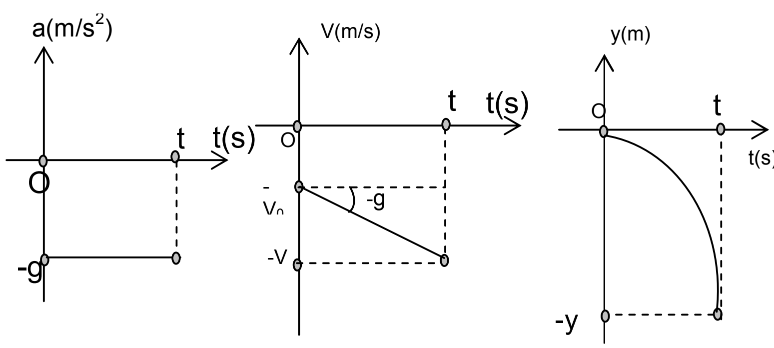
Finally, after 3.00 s (= t_3), the ball's position is

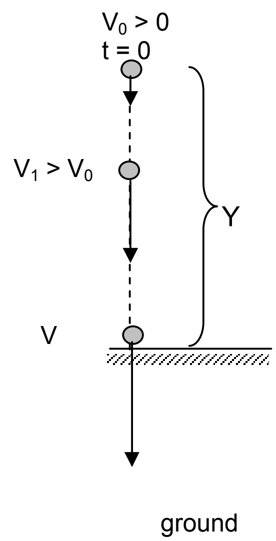
$$y_3 = \frac{1}{2}at_3^2 = \frac{1}{2}(9.80\,\text{m/s}^2)(3.00\,\text{s})^2 = 44.1\,\text{m}$$

OBJECTS THROWN DOWNWARD

If an object is vertically thrown downward from a certain height above the ground with an initial velocity V_0 , it strikes the ground with a velocity bigger than its initial velocity .

If we choose the point where the object is projected as the origin , the a-t , v-t and x-t graphs of this object are shown below ;





line with acceleration and derived the following formulas:

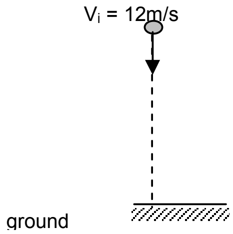
$$V^2 = V_o^2 + 2ax$$
$$V = V_0 + at$$
$$x = V_0 t + \frac{1}{2}at^2$$
$$X = \frac{V_0 + V}{2}t$$

- As we said before objects only under the influence of the gravitational acceleration (g =10 m/s^2) so lets replace the acceleration "a" in these formulas by "g"
- Since object is falling down along the y plane so lets replace "x" by "y"

So the new formulas for the free fall only becomes as:

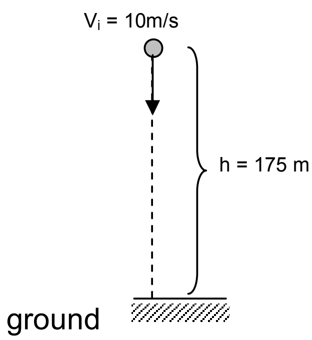
$$V^2 = V_o^2 + 2\,g\,y$$
$$V = V_0 + g\,t$$
$$y = V_o t + \frac{1}{2}g\,t^2$$
$$y = \frac{V_0 + V}{2}t$$

Example1:



An object is vertically projected downward with a velocity of 12 m/s as shown in the figure . It strikes the ground 4 seconds later .
a) What is its velocity just before it strikes the ground ?
b) What is its initial height ?
c) What is its velocity when it is at a height of 47 m above the ground
d) What is its height from the ground 2 seconds later ?

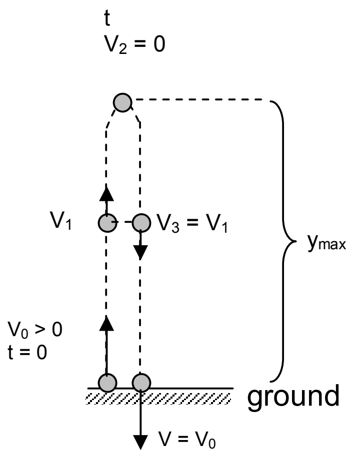
Example 2:



An object is vertically projected downward with a velocity of 10 m/s as shown in the figure . It is initially at a height of 175 m above the ground .

a) What is its velocity just before it strikes the ground ?
b) How long does it take the object to strike the ground ?
c) What is its velocity when it is at a height of 135 m above the ground
d) What is its height from the ground 3 seconds later ?

OBJECTS THROWN UPWARD



If an object is projected upward with an initial velocity V_0 ($V_0 \neq 0$), it reaches a maximum height (y_{max}) where its final velocity becomes zero in t seconds .

This is an example for a uniformly decelerated linear motion. Then , it falls down , and reaches its initial level with the same initial velocity ($V_4 = V_0$) in opposite direction in the next t seconds . This part is simply free fall since the initial velocity is zero.

If we choose the point where the object is projected as the origin , and upward direction as +y axis, in this case acceleration,g, is always directed along the –y axis so g will be minus and our equations for an object thrown upward becomes as following:

http://departments.weber.edu/physics/amiri/director/dcrfiles/projectile/ballMachineOneS.dcr

$$V^2 = V_o^2 - 2\,g\,y$$
$$V = V_0 - g\,t$$
$$y = V_o t - \frac{1}{2} g\,t^2$$
$$y = \frac{V_0 + V}{2}\boldsymbol{t}$$

Now lets try to find out the time (t_{max}) needed to reach the maximum height, y_{max}. Since at the top point velocity (V) of the object becomes zero for a very short period of time, using the second equation given in the box above:

$$V = V_0 - g\,t_{max}$$
$$0 = V_0 - g\,t_{max}$$

$$t_{max} = \frac{V_0}{g}$$

Time needed to reach the Maximum height

$$t_{flight} = 2t_{max} \Rightarrow \quad t_{flight} = 2\frac{V_0}{g}$$

Time of flight.

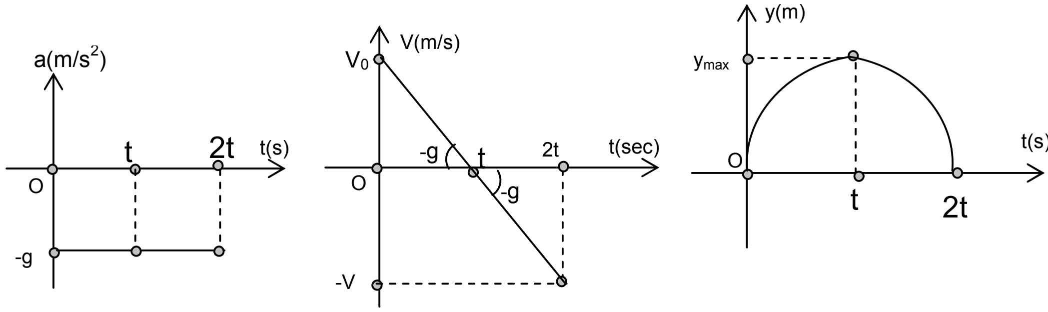
İf we want to find out the max height in terms of the initial velocity V_0

$$y_{max} = V_0 t_{max} - \frac{1}{2} g t^2_{max}$$
$$y_{max} = V_0 (\frac{V_0}{g}) - \frac{1}{2} g (\frac{V_0}{g})^2 \Rightarrow$$
$$y_{max} = \frac{V_0^2}{g} - \frac{1}{2} g \frac{V_0^2}{g^2}$$

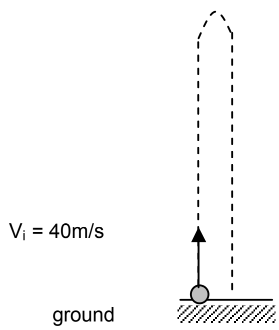
$$y_{max} = \frac{V_0^2}{2g}$$

Maximum height

the a-t , v-t and x-t graphs of this object are shown below ;



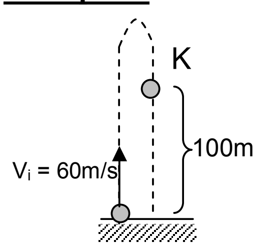
Example 3:



An object is vertically projected upward with a velocity of 40 m/s as shown in the figure .

a) How long does it take the object to reach the maximum height ?
b) What is its time of flight ?
c) What maximum height can the object reach ?
d) What is its height from the ground 6 seconds later ?
e) What is its velocity 3 seconds later ?
f) What is its velocity 7 seconds later ?

Example 4:



An object is vertically projected upward with a velocity of 60 m/s as shown in the figure .
a) How long does it take the object to reach the level K ?
b) What is its velocity at the level K ?

Example 2-11

Suppose the ball in example 2-10 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped? (a) What then would be its position after 1.00 s and 2.00 s? (b) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

Solution:

(a) At t = 1.00 s, the position of the ball is

$$y = v_0 t + \frac{1}{2}at^2 = (3.00\,\text{m/s})(1.00\,\text{s}) + \frac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 7.90\,\text{m}$$

At t = 2.00 s, (time interval t = 0 to t = 2.00 s), the position is

$$y = v_0 t + \frac{1}{2}at^2 = (3.00\,\text{m/s})(2.00\,\text{s}) + \frac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 25.6\,\text{m}$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.

(b) $v = v_0 + at$ = 3.00 m/s + (9.80 m/s^2)(1.00 s) = 12.8 m/s [at t_1 = 1.00 s]
= 3.00 m/s + (9.80 m/s^2)(2.00 s) = 22.6 m/s [at t_2 = 2.00 s]

When the ball was dropped (v_0 = 0), the first term (v_0) in these equation was zero, so
$v = 0 + at$ = (9.80 m/s^2)(1.00 s) = 9.80 m/s [at t_1 = 1.00 s]
= (9.80 m/s^2)(2.00 s) = 19.6 m/s [at t_2 = 2.00 s]

Example 2-12

A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate (a) how high it goes, and (b) how log the ball is in the air before it comes back to his hand.

Solution:
At t = 0 we have y_0 = 0, v_0 = 15.0 m/s, and a = –9.80 m/s^2.
At time t (maximum height), v = 0, a = –9.80 m/s^2, and we wish to find y.

$$y = v_0 t + \frac{1}{2}at^2$$

$$0 = (15.0\,\text{m/s})t + \frac{1}{2}(-9.80\,\text{m/s}^2)t^2$$

This equation is readily factored (we factor out one t): (15.0 m/s – 4.90 m/s$^2 t$)t = 0
There are two solutions:

$$t = 0 \text{ and } t = \frac{15.0\,\text{m/s}}{4.90\,\text{m/s}^2} = 3.06\,\text{s}$$

Conceptual example 2-13

Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, ad (2) that an object thrown upward has zero acceleration at the highest point.

Solution:
Both are wrong. (1) Velocity and acceleration are not necessarily in the same direction. When the object is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point, the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc point upward, then becomes zero (for zero time) at the

highest point, and then points downward. Gravity does not stop acting, so a = –g = –9.80 m/s^2 even there. Thinking that a = 0 at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration (= rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. In sum, the acceleration of gravity always points down toward the Earth, even when the object is moving up.

Example 2-14

Let us consider again the ball thrown upward of example 2-12, and make more calculations. Calculate (a) how much time it takes for the ball to reach the maximum height, and (b) the velocity of the ball when it returns to the thrower's hand.

Solution:

(a) $a = -9.80\ \text{m/s}^2$, $v_0 = 15.0$ m/s, and $v = 0$:
$v = v_0 + at$;
setting $v = 0$ and solving for t gives

$$t = -\frac{v_0}{a} = -\frac{15.0\,\text{m/s}}{-9.80\,\text{m/s}^2} = 1.53\,\text{s}$$

(b) $t = 3.06$ s:
$v = v_0 + at$ -= 15.0 m/s – (9.80 m/s^2)(3.06 s) = –15.0 m/s.

Example 2-15

For the ball in example 2-14, calculate at what time t the ball passes a point 8.00 m above the person's hand.

Solution:
$y = 8.00$ m, $y_0 = 0$, $v_0 = 15.0$ m/s and $a = -9.80\ \text{m/s}^2$.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

$$8.00\,\text{m} = 0 + (15.0\,\text{m/s})t + \frac{1}{2}(-9.80\,\text{m/s}^2)t^2$$

To solve any quadratic equation of the form at^2 + bt + c = 0, where a, b and c are constant (a is not acceleration here), we use the quadratic formula:

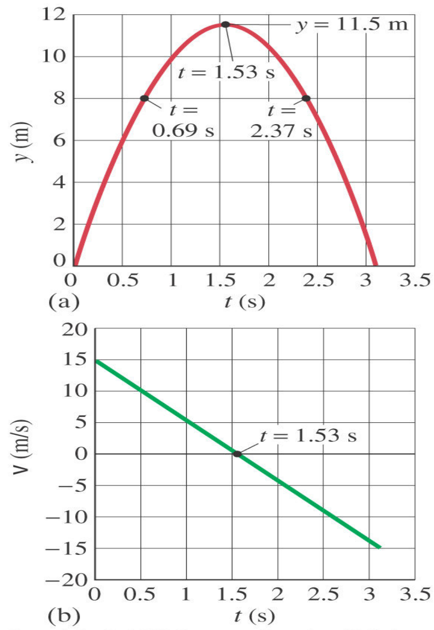
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\,\text{m/s}^2)t^2 - (15.0\,\text{m/s})t + (8.00\,\text{m}) = 0.$$

$$t = \frac{15.0\,\text{m/s} \pm \sqrt{(15.0\,\text{m/s})^2 - 4(4.90\,\text{m/s}^2)(8.00\,\text{m})}}{2(4.90\,\text{m/s}^2)}$$ which gives us $t = 0.69$ s and $t = 2.37$ s.

The Figure below shows theGraphs of (a) y vs. t, (b) v vs. t for a ball thrown upward. Examples 2–12, 2–14, and 2–15.



<u>Now answer the following questions:</u>

1. A stone is dropped from the top of a cliff. It hits the below after 3.25 s. How high is the cliff?

2. If a car rolls gently ($v_0 = 0$) off a vertical cliff, how long does it take it to reach 85 km/h?

3. Estimate (a) how long its took King Kong to fall straight down from the top of the Empire State building (380 m high), and (b) his velocity just before "landing" ?

4. A baseball is hit nearly straight up into the air with a speed of 22 m/s. (a) How high does it go? (b) How long is it in the air?

5. A ballplayer catches a hall 3.0s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

6. An object starts from rest and falls under the influence of gravity. Draw graphs of

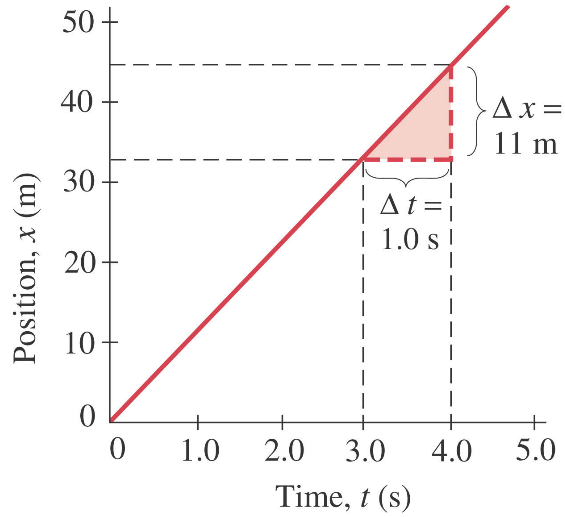
(a) its speed and

(b) the distance it bas fallen, as a function of time from $t = 0$ to $t = 5.00$ s. (Ignore air resistance.)

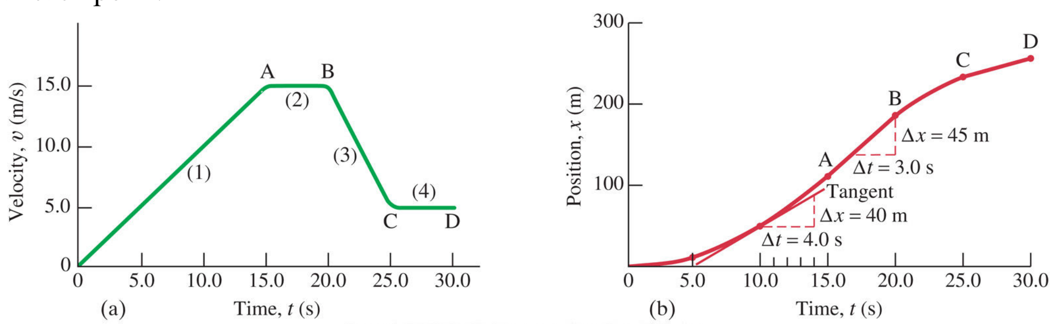
7. A helicopter is ascending vertically with a speed of 5.20 m/s. At a height of 125 m above the Earth, a package is dropped from a window. How much time does it take for the package to reach the ground? [Hint: The package's initial speed equals the helicopter's.]

2-8 Graphical Analysis of Linear Motion

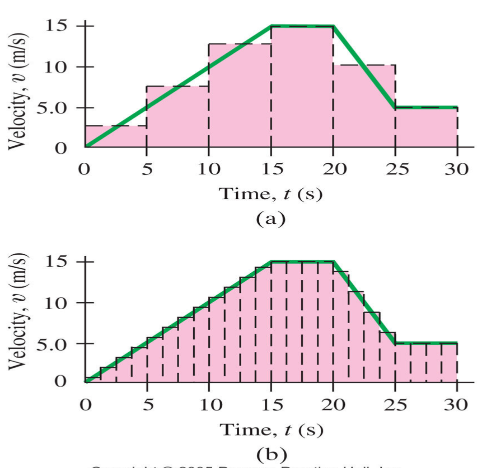
This is a graph of x vs. t for an object moving with constant velocity. The velocity is the slope of the x-t curve.



On the left we have a graph of velocity vs. time for an object with varying velocity; on the right we have the resulting x vs. t curve. The instantaneous velocity is tangent to the curve at each point.



The displacement, x, is the area beneath the v vs. t curve.



Example 2-16

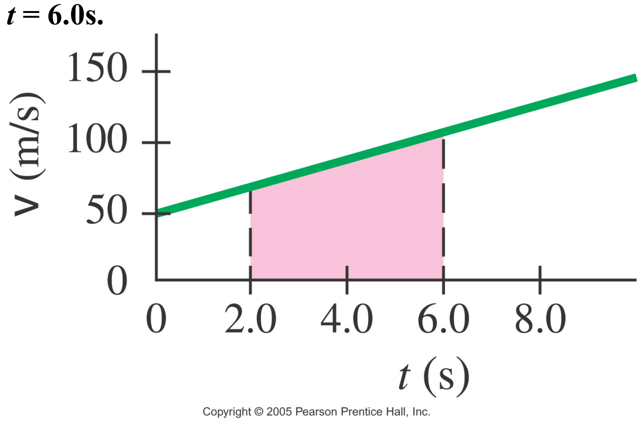
A space probe accelerates uniformly from 50 m/s at t = 0 to 150 m/s at t = 10 s. How far did it move between t = 2.0 s and t = 6.0 s?

Solution:

The acceleration a = (150 m/s – 50 m/s)/10 s = 10 m/s^2.
At t = 2.0 s, v = 70 m/s and at t = 6.0 s, v = 110 m/s.
Thus the area, $(\bar{v}\times\Delta t)$, which equals Δx, is

$$\Delta x = \left(\frac{70\,\text{m/s} + 110\,\text{m/s}}{2}\right)(4.0\,\text{s}) = 360\,\text{m}$$

The shaded area represents the displacement during the time interval t = 2.0s to t = 6.0s.



Problems For Extra Practise:

1. A rolling ball moves from x_1 = 3.4 cm to x_2 = -4.2 cm during the time from t_1 = 3.0 s to t_2 = 6.1 s. What is its average velocity?

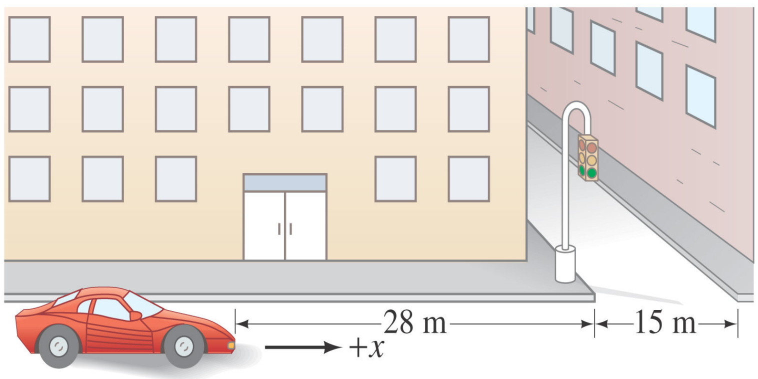
2. A particle at t_1 = –2.0 s is at x_1 = 3.4 cm and at t_2 = 4.5 s is at x_2 = 8.5 cm. What is its average velocity? Can you calculate its average speed from these data?

3. You are driving home from school steadily at 95 km/h for 130 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 3 hours and 20minutes.
(a) How far is your hometown from school?
(b) What was your average speed?

4. A car is behind a truck going 25 m/s on the highway. The car's driver looks for an opportunity to pass, guessing that his car can accelerate at 1.0 m/s^2. He gauges that he has to cover the 20-m length of the truck, plus 10 m clear room at the rear of the truck and 10 m more at the front of it. In the oncoming lane, he sees a car approaching, probably also traveling at 25 m/s. He estimates that the car is about 400 m away. Should he attempt the pass? Give details.

5. A runner hopes to complete the 10,000-m run in less than 30.0 min. After exactly 27.0 min, there are still1100 m to go. The runner must then accelerate at 0.20 m/s^2 for how many seconds in order to achieve the desired time?

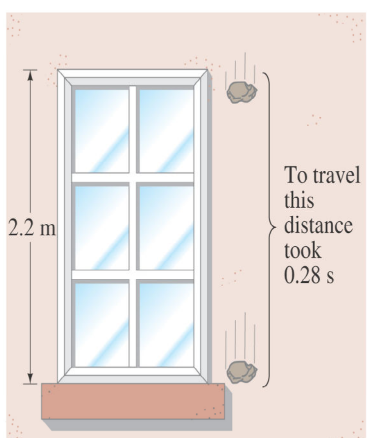
6. A person driving her car at 45 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning red, and she is 28 m away from the near side of the intersection (Fig. 2-31). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide: Her car's maximum deceleration is $-5.8\ m/s^2$, whereas it can accelerate from 45 km/h to 65 km in 6.0 s. Ignore the length of her car and her reaction time.



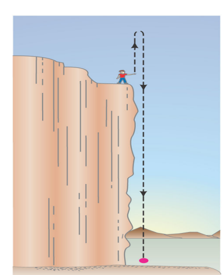
7. A stone is thrown vertically upward with a speed of 18.0 m/s. (a) How fast is it moving when it reaches a height of 11.0m? (b) How long is required to reach this height? (c) Why are there two answers to (b)?

8. A falling stone takes 0.28 s to travel past a window 2.2 m tall (figure below). From what height above the top of the window did the stone fall?

Problem 8

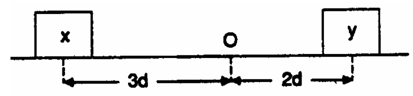


9. A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff
70.0 m high (figure below).
(a) How much later does it reach the bottom of the cliff?
(b) What is its speed just before hitting?
(c) What total distance did it travel?

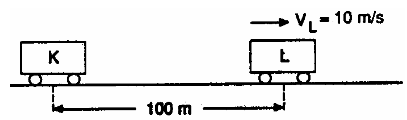


10. A baseball is seen to pass upward by a window 28 m above the street with a vertical speed of 13 m/s. If the ball was thrown from the street, (a) what was it initial speed, (b) what altitude does it reach, (c) when was it thrown, and (d) when does it reach the street again?

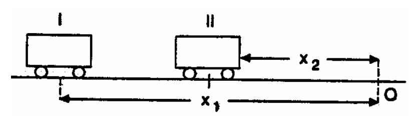
11. The objects x and y are initially at rest. They start accelerating at a constant rate at the same time toward each other. They meet at the point O. What is the ratio of the magnitudes of their accelerations ?



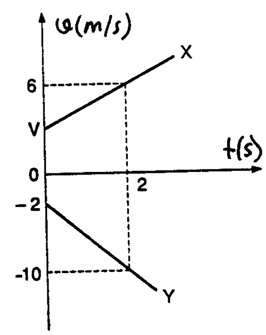
12. The object K is initially at rest and 100m behind the object L. It starts speeding up at a constant acceleration of $4 m/s^2$ while the object L moves at a constant velocity of 10m/s as shown in the figure above. At what time can the object K catch up the object L



13. The objects I and II are initially at rest. The object I speeds up at a constant acceleration of 2a and the object II speeds up at a constant acceleration of a. They travel x_1 and x_2 distances until they arrive at the point O.
What is the ratio , x_1 / x_2 ?



14. The moving objects X and Y are side by side at t=0 s. Their velocity-time graphs are as shown in the figure above. If the distance between them is 22m after t=2 s seconds, what is the acceleration of the moving object X in terms m/s^2?



15. The moving objects K and L have velocity-time graphs as shown in Fig.2. above when they are 60m away from each other at t=0s. What was the distance between them in m at the end of 4s ?

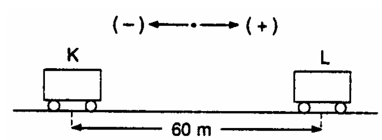


Fig.1.

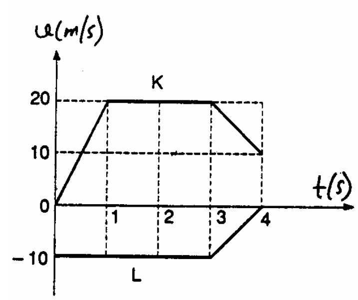


Fig.2.

Summary of Chapter 2

- Kinematics is the description of how objects move with respect to a defined reference frame.
- Displacement is the change in position of an object.
- Average speed is the distance traveled divided by the time it took; average velocity is the displacement divided by the time.
- Instantaneous velocity is the limit as the time becomes infinitesimally short.
- Average acceleration is the change in velocity divided by the time.
- Instantaneous acceleration is the limit as the time interval becomes infinitesimally small.
- The equations of motion for constant acceleration are given in the text; there are four, each one of which requires a different set of quantities.
- Objects falling (or having been projected) near the surface of the Earth experience a gravitational acceleration of 9.80 m/s^2.